

Distance Functions and Metric Learning: Part 1

Michael Werman Ofir Pele

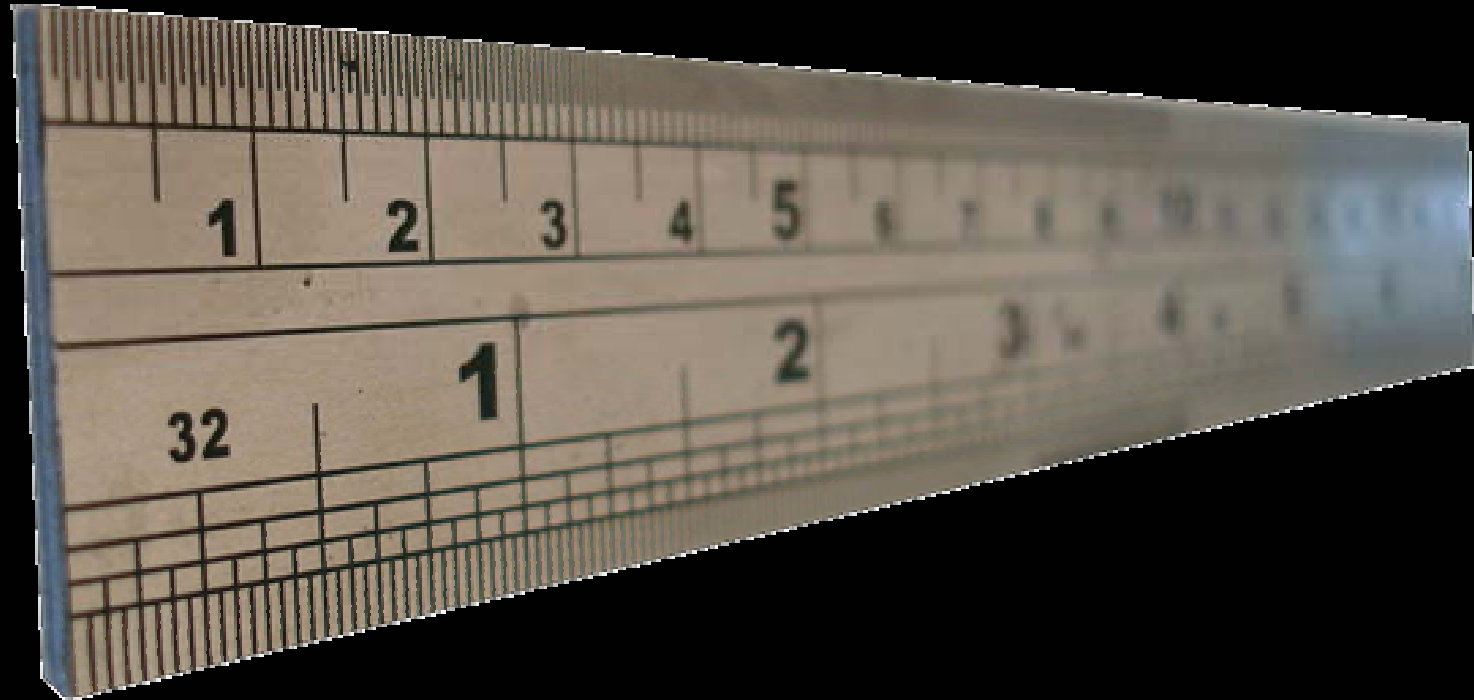


The Hebrew University
of Jerusalem

Rough Outline

- Bin-to-Bin Distances.
- Cross-Bin Distances:
 - Quadratic-Form (aka Mahalanobis) / Quadratic-Chi.
 - The Earth Mover's Distance.
- Perceptual Color Differences.
- Hands-On Code Example.

Distance ?



Metric

- Non-negativity:

$$D(P, Q) \geq 0$$

- Identity of indiscernibles:

$$D(P, Q) = 0 \text{ iff } P = Q$$

- Symmetry:

$$D(P, Q) = D(Q, P)$$

- Subadditivity (triangle inequality):

$$D(P, Q) \leq D(P, K) + D(K, Q)$$

Pseudo-Metric (aka Semi-Metric)

- Non-negativity:

$$D(P, Q) \geq 0$$

- Property changed to:

$$D(P, Q) = 0 \text{ if } P = Q$$

- Symmetry:

$$D(P, Q) = D(Q, P)$$

- Subadditivity (triangle inequality):

$$D(P, Q) \leq D(P, K) + D(K, Q)$$

Metric

- Non-negativity:

$$D(P, Q) \geq 0$$

- Identity of indiscernibles:

$$D(P, Q) = 0 \text{ iff } P = Q$$

An object is most similar to itself

Metric

- Symmetry:

$$D(P, Q) = D(Q, P)$$

- Subadditivity (triangle inequality):

$$D(P, Q) \leq D(P, K) + D(K, Q)$$

Useful for many algorithms

Minkowski-Form Distances

$$L_p(P, Q) = \left(\sum_i |P_i - Q_i|^p \right)^{\frac{1}{p}}$$

Minkowski-Form Distances

$$L_1(P, Q) = \sum_i |P_i - Q_i|$$

$$L_2(P, Q) = \sqrt{\sum_i (P_i - Q_i)^2}$$

$$L_\infty(P, Q) = \max_i |P_i - Q_i|$$

Kullback-Leibler Divergence

$$KL(P, Q) = \sum_i P_i \log \frac{P_i}{Q_i}$$

- Information theoretic origin.
- Non symmetric.
- $Q_i = 0$?

Jensen-Shannon Divergence

$$JS(P, Q) = \frac{1}{2}KL(P, M) + \frac{1}{2}KL(Q, M)$$

$$M = \frac{1}{2}(P + Q)$$

$$JS(P, Q) = \frac{1}{2} \sum_i P_i \log \frac{2P_i}{P_i + Q_i} + \frac{1}{2} \sum_i Q_i \log \frac{2Q_i}{P_i + Q_i}$$

Jensen-Shannon Divergence

$$JS(P, Q) = \frac{1}{2}KL(P, M) + \frac{1}{2}KL(Q, M)$$

$$M = \frac{1}{2}(P + Q)$$

- Information Theoretic origin.
- Symmetric.
- $\sqrt{JS}$ is a metric.

Jensen-Shannon Divergence

- Using Taylor extension and some algebra:

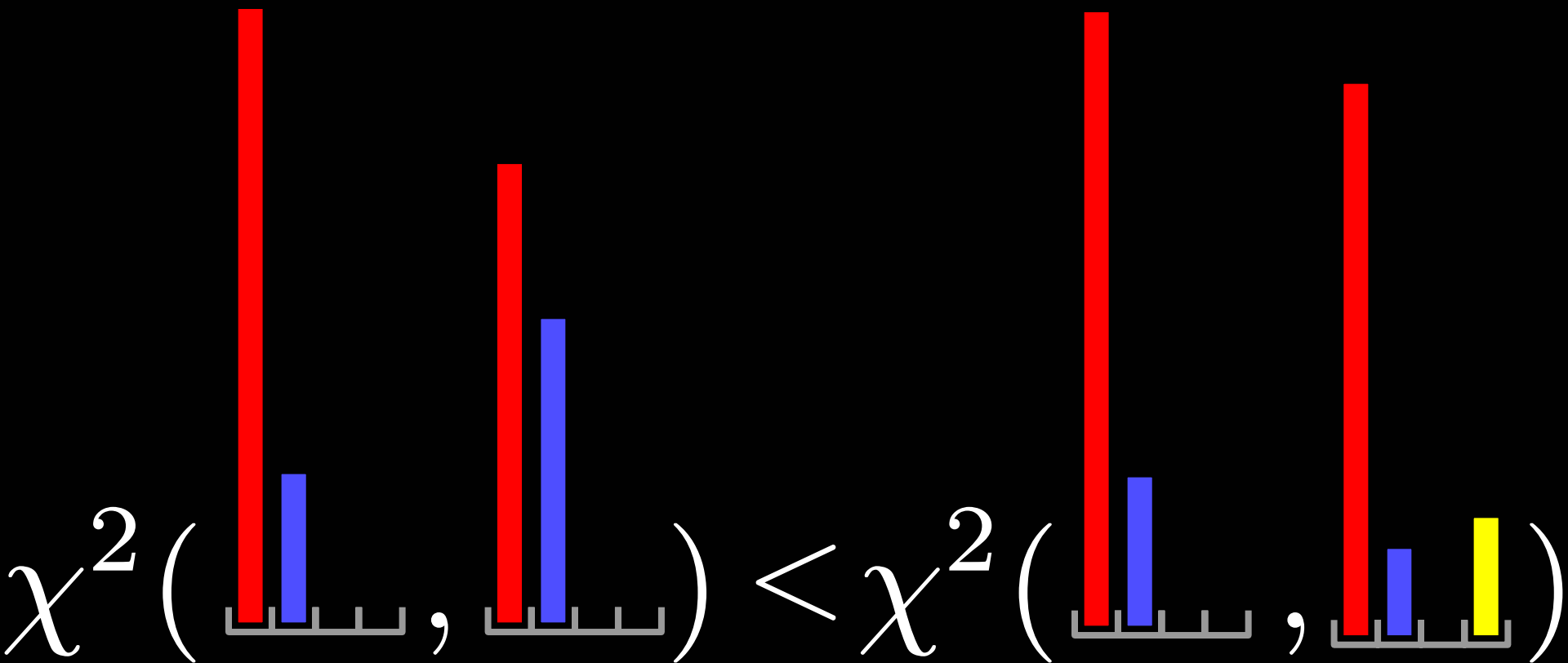
$$\begin{aligned} JS(P, Q) &= \sum_{n=1}^{\infty} \frac{1}{2n(2n-1)} \sum_i \frac{(P_i - Q_i)^{2n}}{(P_i + Q_i)^{2n-1}} \\ &= \frac{1}{2} \sum_i \frac{(P_i - Q_i)^2}{(P_i + Q_i)} + \\ &\quad \frac{1}{12} \sum_i \frac{(P_i - Q_i)^4}{(P_i + Q_i)^3} + \dots \end{aligned}$$

χ^2 Histogram Distance

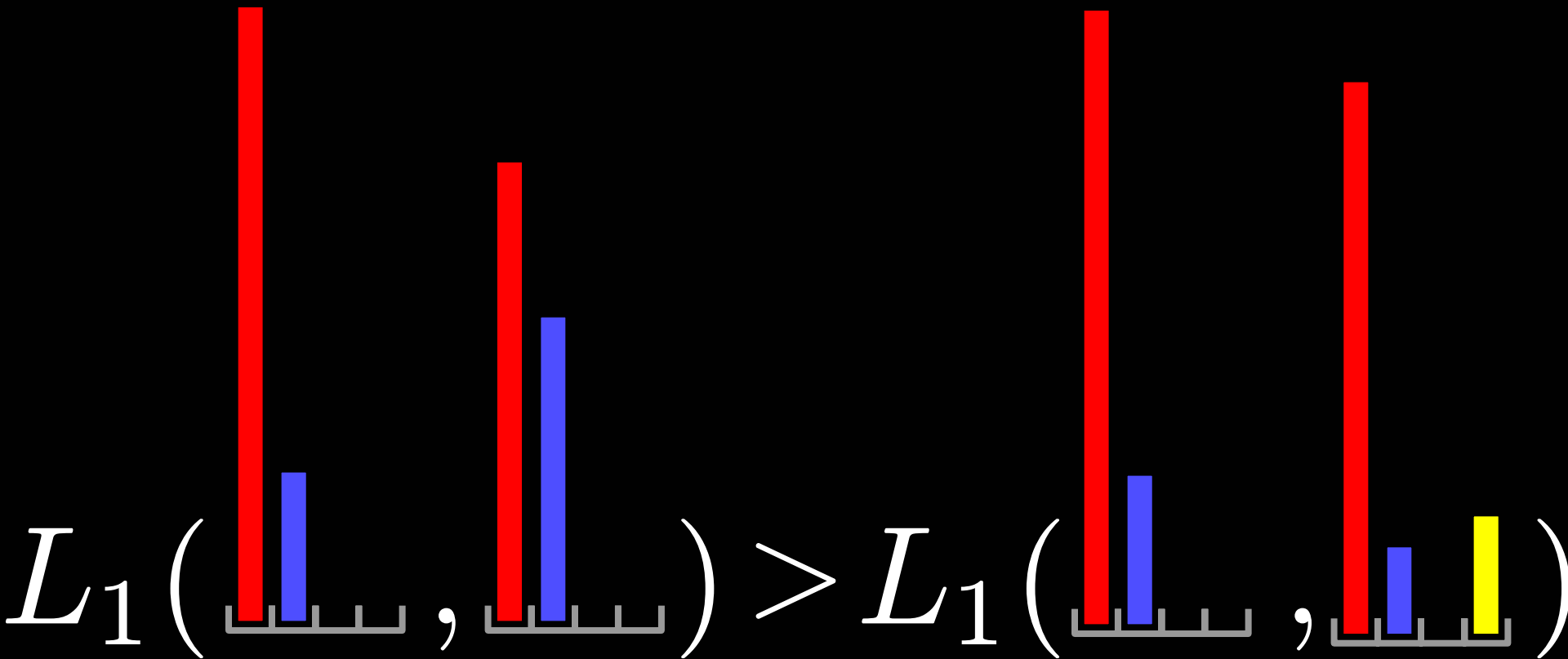
$$\chi^2(P, Q) = \frac{1}{2} \sum_i \frac{(P_i - Q_i)^2}{(P_i + Q_i)}$$

- Statistical origin.
- Experimentally results are very similar to JS.
- Reduces the effect of large bins.
- $\sqrt{\chi^2}$ is a metric

χ^2 Histogram Distance



χ^2 Histogram Distance



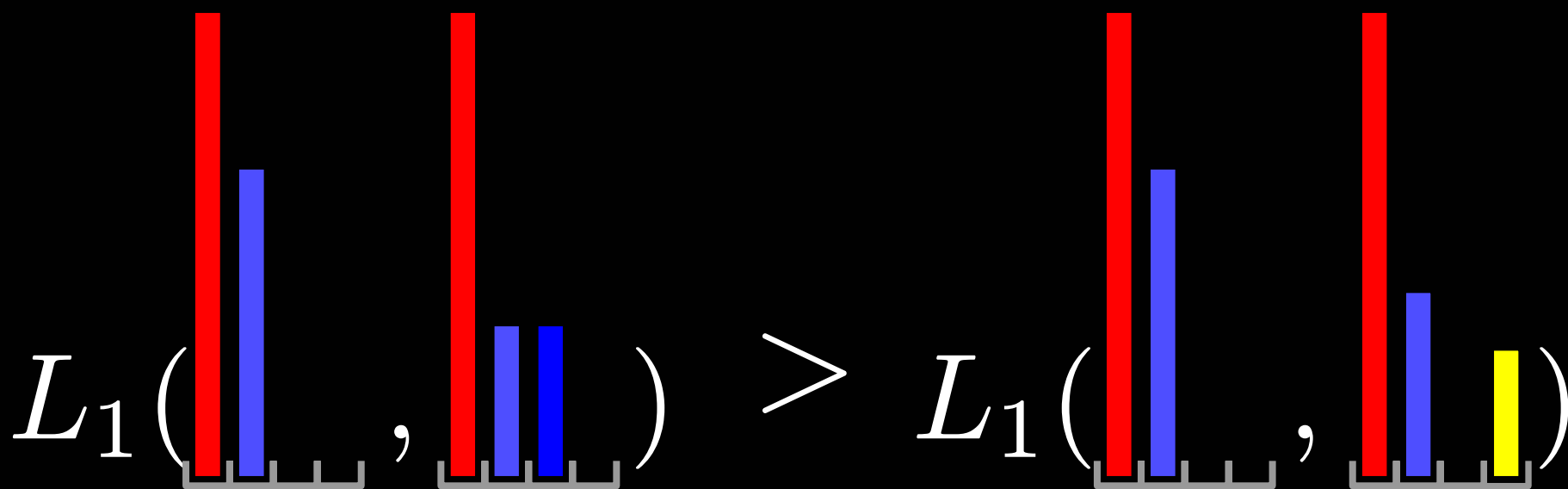
χ^2 Histogram Distance

$$\chi^2(P, Q) = \frac{1}{2} \sum_i \frac{(P_i - Q_i)^2}{(P_i + Q_i)}$$

- Experimentally better than L_2 .

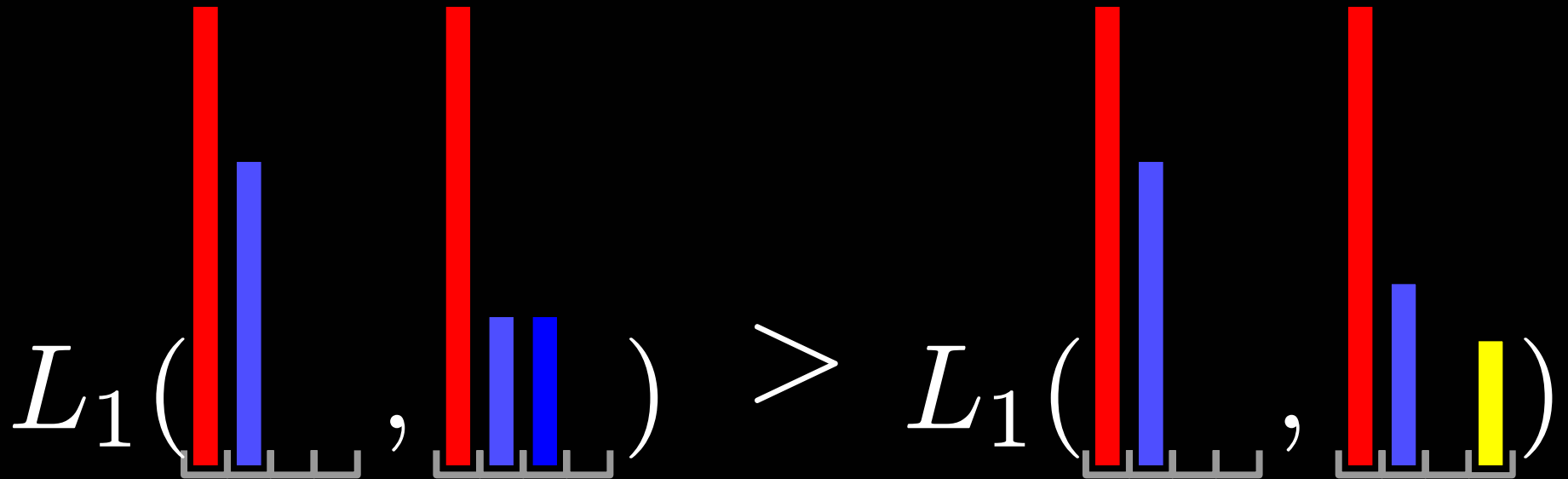
Bin-to-Bin Distances

- Bin-to-Bin distances such as L_1 , L_2 , χ^2 are sensitive to quantization:



Bin-to-Bin Distances

- #bins  $\Rightarrow$ robustness  distinctiveness 
- #bins  $\Rightarrow$ robustness  distinctiveness 



Bin-to-Bin Distances

- #bins  $\Rightarrow$ robustness  distinctiveness 
- #bins  $\Rightarrow$ robustness  distinctiveness 

Can we achieve
robustness
and
distinctiveness ?

The Quadratic-Form Histogram Distance

$$QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)}$$

$$= \sqrt{\sum_{ij} (P_i - Q_i)(P_j - Q_j) A_{ij}}$$

- A_{ij} is the similarity between bin i and j .
- If A is the inverse of the covariance matrix, QF is called Mahalanobis distance.

The Quadratic-Form Histogram Distance

$$QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)}$$

$$= \sqrt{\sum_{ij} (P_i - Q_i)(P_j - Q_j) A_{ij}}$$

$$A = I$$

$$\Rightarrow \sqrt{\sum_{ij} (P_i - Q_i)^2} = L_2(P, Q)$$

The Quadratic-Form Histogram Distance

$$QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)}$$

- Does not reduce the effect of large bins.
- Alleviates the quantization problem.
- Linear time computation in # non zero A_{ij} .

The Quadratic-Form Histogram Distance

$$QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)}$$

- If A is **positive-semidefinite** then QF is a **pseudo-metric**.

Pseudo-Metric (aka Semi-Metric)

- Non-negativity:

$$D(P, Q) \geq 0$$

- Property changed to:

$$D(P, Q) = 0 \text{ if } P = Q$$

- Symmetry:

$$D(P, Q) = D(Q, P)$$

- Subadditivity (triangle inequality):

$$D(P, Q) \leq D(P, K) + D(K, Q)$$

The Quadratic-Form Histogram Distance

$$QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)}$$

- If A is **positive-definite** then QF is a metric.

The Quadratic-Form Histogram Distance

$$\begin{aligned} QF^A(P, Q) &= \sqrt{(P - Q)^T A (P - Q)} \\ &= \sqrt{(P - Q)^T W^T W (P - Q)} \\ &= L_2(WP, WQ) \end{aligned}$$

The Quadratic-Form Histogram Distance

$$\begin{aligned} QF^A(P, Q) &= \sqrt{(P - Q)^T A (P - Q)} \\ &= L_2(WP, WQ) \end{aligned}$$

We assume there is a **linear transformation** that makes bins **independent**

There are cases where this is **not true**
e.g. **COLOR**

The Quadratic-Form Histogram Distance

$$QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)}$$

- Converting distance to similarity (Hafner *et. al* 95):

$$A_{ij} = 1 - \frac{D_{ij}}{\max_{ij}(D_{ij})}$$

The Quadratic-Form Histogram Distance

$$QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)}$$

- Converting distance to similarity (Hafner *et. al* 95):

$$A_{ij} = e^{-\alpha \frac{D(i, j)}{\max_{i, j} (D(i, j))}}$$

If α is large enough,
A will be positive-definitive

The Quadratic-Form Histogram Distance

$$QF^A(P, Q) = \sqrt{(P - Q)^T A (P - Q)}$$

- Learning the similarity matrix: part 2 of this tutorial.

The Quadratic-Chi Histogram Distance

$$\text{QC}_m^A(P, Q) = \sqrt{\sum_{ij} \frac{(P_i - Q_i)(P_j - Q_j)A_{ij}}{(\sum_c (P_c + Q_c)A_{ci})^m (\sum_c (P_c + Q_c)A_{cj})^m}}$$

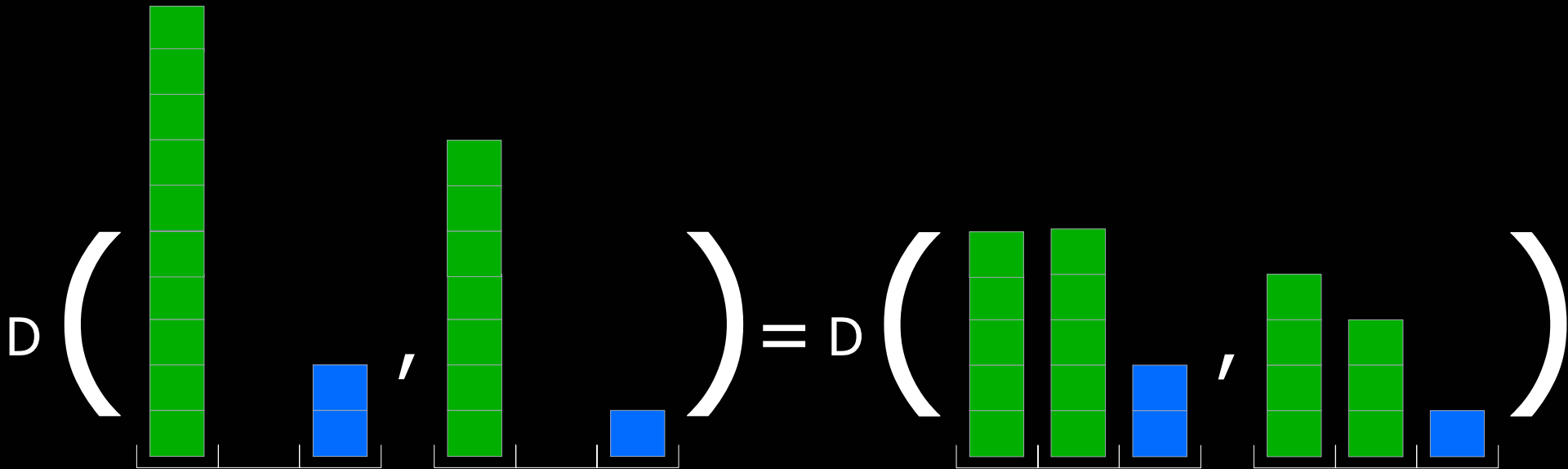
- A_{ij} is the similarity between bin i and j .
- Generalizes QF and χ^2 .
- Reduces the effect of large bins.
- Alleviates the quantization problem.
- Linear time computation in # non zero A_{ij} .

The Quadratic-Chi Histogram Distance

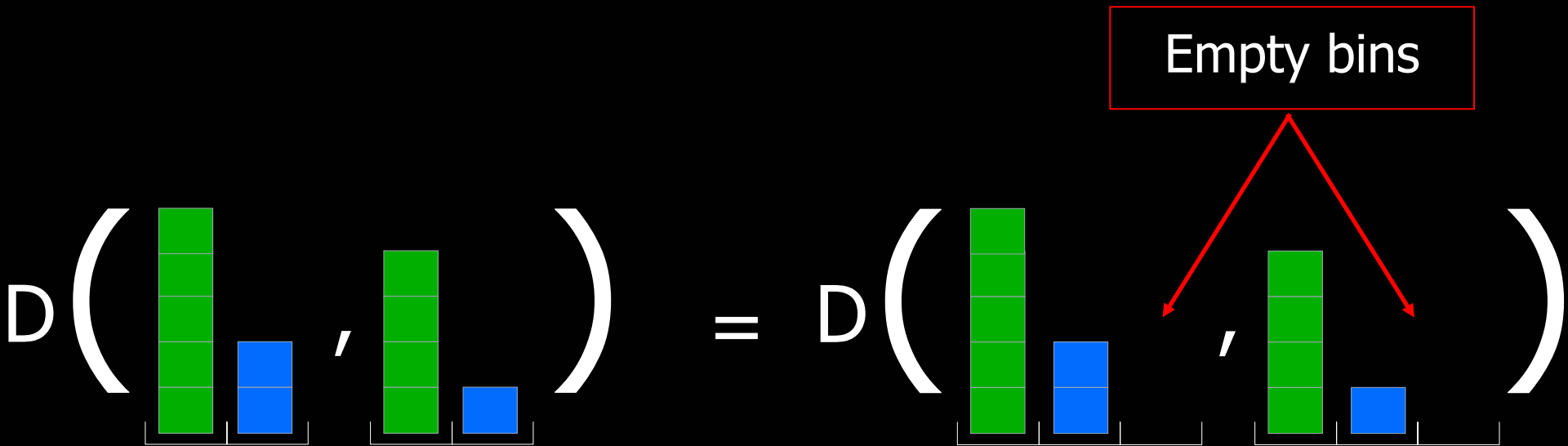
$$QC_m^A(P, Q) = \sqrt{\sum_{ij} \frac{(P_i - Q_i)(P_j - Q_j)A_{ij}}{(\sum_c (P_c + Q_c)A_{ci})^m (\sum_c (P_c + Q_c)A_{cj})^m}}$$

- is non-negative if A is positive-semidefinite.
- Symmetric.
- Triangle inequality unknown.
- If we define $\frac{0}{0} = 0$ and $0 \leq m < 1$, QC is continuous.

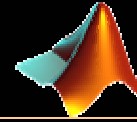
Similarity-Matrix-Quantization-Invariant Property



Sparseness-Invariant



The Quadratic-Chi Histogram Distance

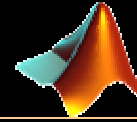


Code

```
function dist= QuadraticChi(P,Q,A,m)

Z= (P+Q)*A;
% 1 can be any number as Z_i==0 iff D_i=0
Z(Z==0)= 1;
Z= Z.^m;
D= (P-Q)./Z;
% max is redundant if A is
% positive-semidefinite
dist= sqrt( max(D*A*D',0) );
```

The Quadratic-Chi Histogram Distance



Code

Time Complexity: $O(\#A_{ij} \neq 0)$

```
function dist= QuadraticChi(P,Q,A,m)

Z= (P+Q)*A;
% 1 can be any number as Z_i==0 iff D_i=0
Z(Z==0)= 1;
Z= Z.^m;
D= (P-Q)./Z;
% max is redundant if A is
% positive-semidefinite
dist= sqrt( max(D*A*D',0) );
```

The Quadratic-Chi Histogram Distance Code

What about sparse (e.g. BoW) histograms ?

```
function dist= QuadraticChi(P,Q,A,m)

Z= (P+Q)*A;
% 1 can be any number as Z_i==0 iff D_i=0
Z(Z==0)= 1;
Z= Z.^m;
D= (P-Q)./Z;
% max is redundant if A is
% positive-semidefinite
dist= sqrt( max(D*A*D',0) );
```

The Quadratic-Chi Histogram Distance Code

What about sparse (e.g. BoW) histograms ?

$$P - Q =$$



Time Complexity: $O(\textcolor{red}{S}\textcolor{green}{K})$

The Quadratic-Chi Histogram Distance Code

What about sparse (e.g. BoW) histograms ?

$$P - Q =$$



Time Complexity: $O(\textcolor{red}{S}\textcolor{green}{K})$

$$\textcolor{red}{\#(P \neq 0)} + \textcolor{red}{\#(Q \neq 0)}$$

The Quadratic-Chi Histogram Distance Code

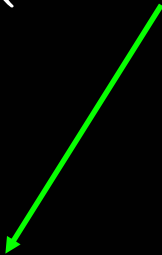
What about sparse (e.g. BoW) histograms ?

$$P - Q =$$



Time Complexity: $O(\textcolor{red}{S}\textcolor{green}{K})$

Average of non-zero entries
in each row of A



The Earth Mover's Distance

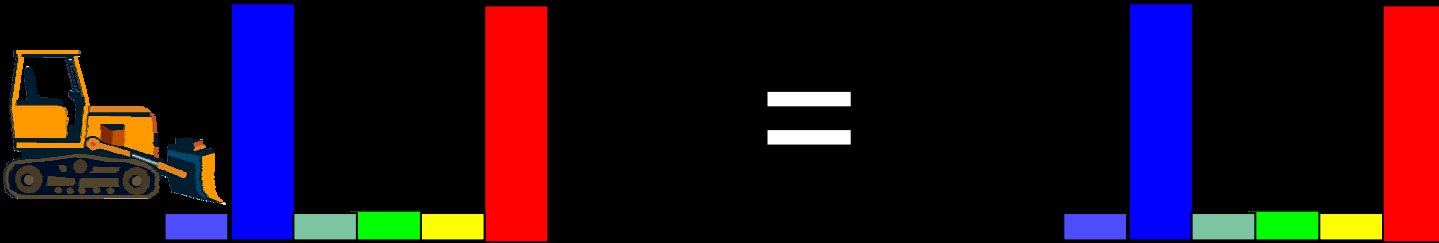
The Earth Mover's Distance

- The Earth Mover's Distance is defined as the **minimal cost** that must be paid to transform one histogram into the other, where there is a “**ground distance**” between the basic features that are aggregated into the histogram.

The Earth Mover's Distance



The Earth Mover's Distance



The Earth Mover's Distance

$$EMD^D(P, Q) = \min_{F=\{F_{ij}\}} \sum_{i,j} F_{ij} D_{ij}$$

$$s.t : \quad \sum_j F_{ij} = P_i \quad \sum_i F_{ij} = Q_j$$

$$\sum_{i,j} F_{ij} = \sum_i P_i = \sum_j Q_j = 1$$

$$F_{ij} \geq 0$$

The Earth Mover's Distance

$$EMD^D(P, Q) = \min_{F=\{F_{ij}\}} \frac{\sum_{i,j} F_{ij} D_{ij}}{\sum_i F_{ij}}$$

$$s.t : \quad \sum_j F_{ij} \leq P_i \quad \sum_i F_{ij} \leq Q_j$$

$$\sum_{i,j} F_{ij} = \min\left(\sum_i P_i, \sum_j Q_j\right)$$

$$F_{ij} \geq 0$$

The Earth Mover's Distance

- Pele and Werman 08 – $\widehat{EMD}$, a new EMD definition.

Definition:

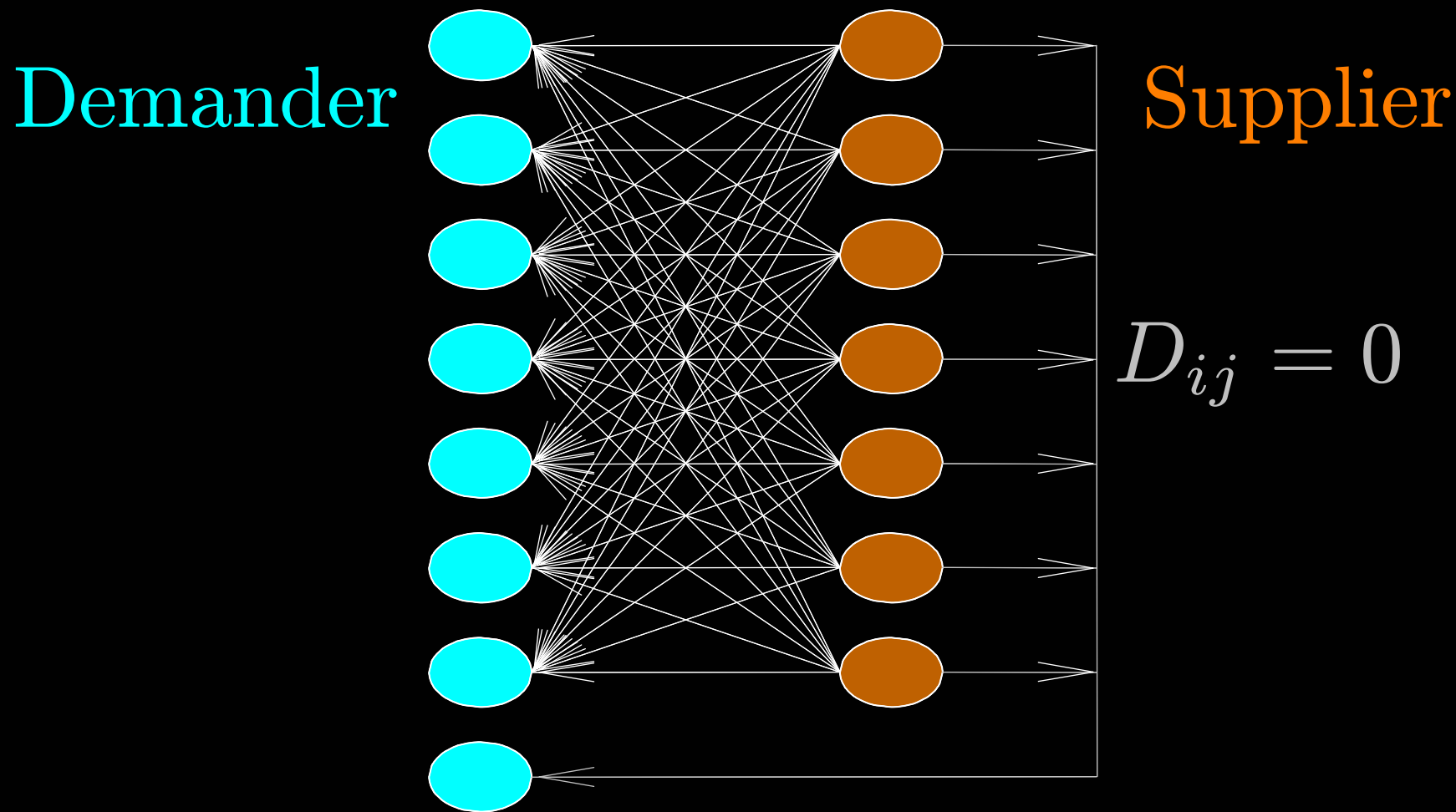
$$\widehat{EMD}_C^D(P, Q) = \min_{F=\{F_{ij}\}} \sum_{i,j} F_{ij} D_{ij} +$$

$$\left| \sum_i P_i - \sum_j Q_j \right| \times C$$

$$s.t : \quad \sum_j F_{ij} \leq P_i \quad \sum_i F_{ij} \leq Q_j$$

$$\sum_{i,j} F_{ij} = \min\left(\sum_i P_i, \sum_j Q_j\right) \quad F_{ij} \geq 0$$

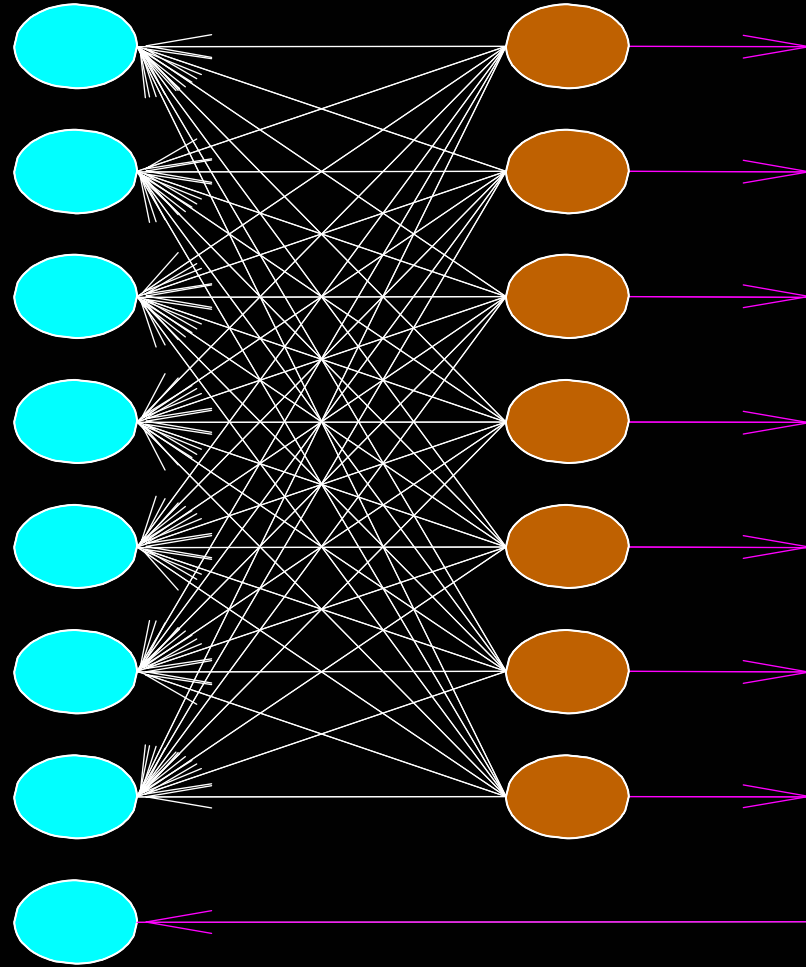
EMD



$$\sum \text{Demander} \leq \sum \text{Supplier}$$

$\widehat{EMD}$

Demander



Supplier

$$D_{ij} = C$$

$$\sum \text{Demander} \leq \sum \text{Supplier}$$

When to Use $\widehat{EMD}$

- When the total mass of two histograms is important.

$$EMD\left(\begin{array}{|c|c|} \hline \text{■} & \text{■} \\ \hline \end{array}, \begin{array}{|c|c|} \hline \text{■} & \text{■} \\ \hline \end{array}\right) = EMD\left(\begin{array}{|c|c|} \hline \text{■} & \text{■} \\ \hline \end{array}, \begin{array}{|c|c|} \hline \text{■} & \text{■} \\ \hline \end{array}\right)$$

When to Use $\widehat{EMD}$

- When the total mass of two histograms is important.

$$EMD(\text{Histogram 1}, \text{Histogram 2}) = EMD(\text{Histogram 1}, \text{Histogram 2})$$

$$\widehat{EMD}(\text{Histogram 1}, \text{Histogram 2}) < \widehat{EMD}(\text{Histogram 1}, \text{Histogram 2})$$

When to Use $\widehat{EMD}$

- When the difference in total mass between histograms is a distinctive cue.

$$EMD(\text{■ ■}, \text{■ ■ ■}) = EMD(\text{■ ■}, \text{■ ■ ■ ■}) = 0$$

When to Use $\widehat{EMD}$

- When the difference in total mass between histograms is a distinctive cue.

$$EMD(\text{■ ■}, \text{■ ■}) = EMD(\text{■ ■}, \text{■ ■}) = 0$$

$$\widehat{EMD}(\text{■ ■}, \text{■ ■}) < \widehat{EMD}(\text{■ ■}, \text{■ ■})$$

When to Use $\widehat{EMD}$

- If ground distance is a metric:
- EMD is a metric only for **normalized** histograms.
- $\widehat{EMD}$ is a metric for **all** histograms ($C \geq \frac{1}{2}\Delta$).

$\widehat{EMD}$ - a Natural Extension to L_1

- $\widehat{EMD} = L_1$ if:

$$D_{ij} = \begin{cases} 0 & \text{if } i = j \\ 2 & \text{otherwise} \end{cases} \quad C = 1$$

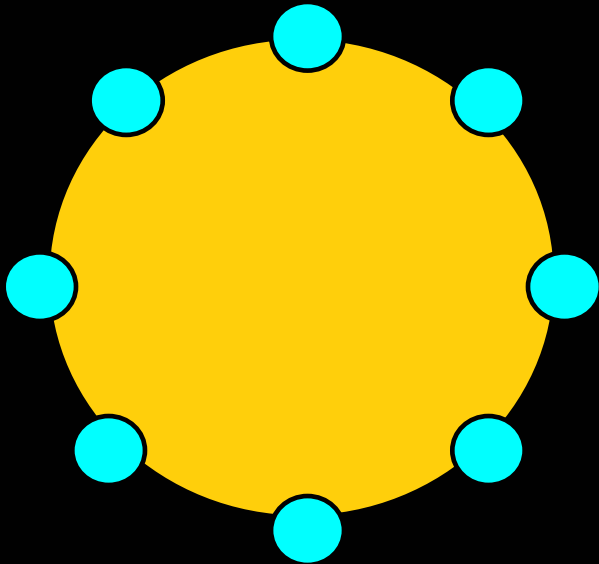
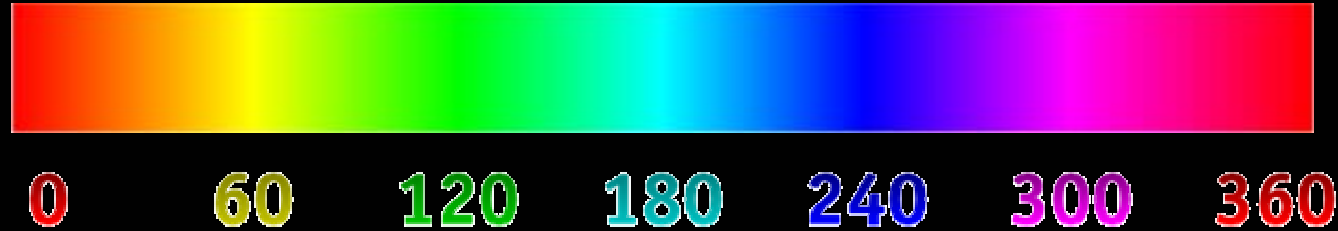
$$\widehat{EMD}_C^D(P, Q) = \min_{F=\{F_{ij}\}} \sum_{i,j} F_{ij} D_{ij} + \left| \sum_i P_i - \sum_j Q_j \right| \times C$$

The Earth Mover's Distance Complexity Zoo

- General ground distance: $O(N^3 \log N)$
 - Orlin 88
- L_1 normalized 1D histograms $O(N)$
 - Werman, Peleg and Rosenfeld 85

The Earth Mover's Distance Complexity Zoo

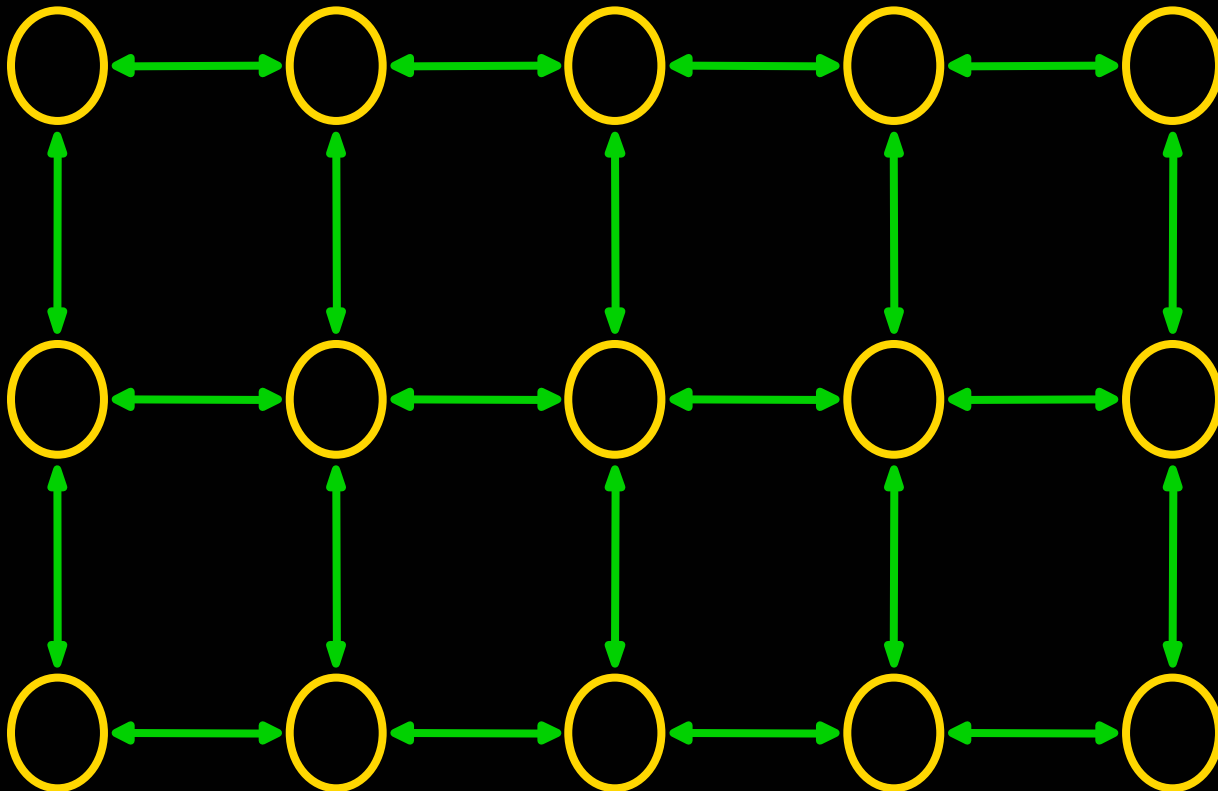
- L_1 normalized 1D cyclic histograms $O(N)$
 - Pele and Werman 08 (Werman, Peleg, Melter, and Kong 86)



*	+	→	→
↗	↖	↗	↗
↗	←	→	→
↑	↑	↗	↗

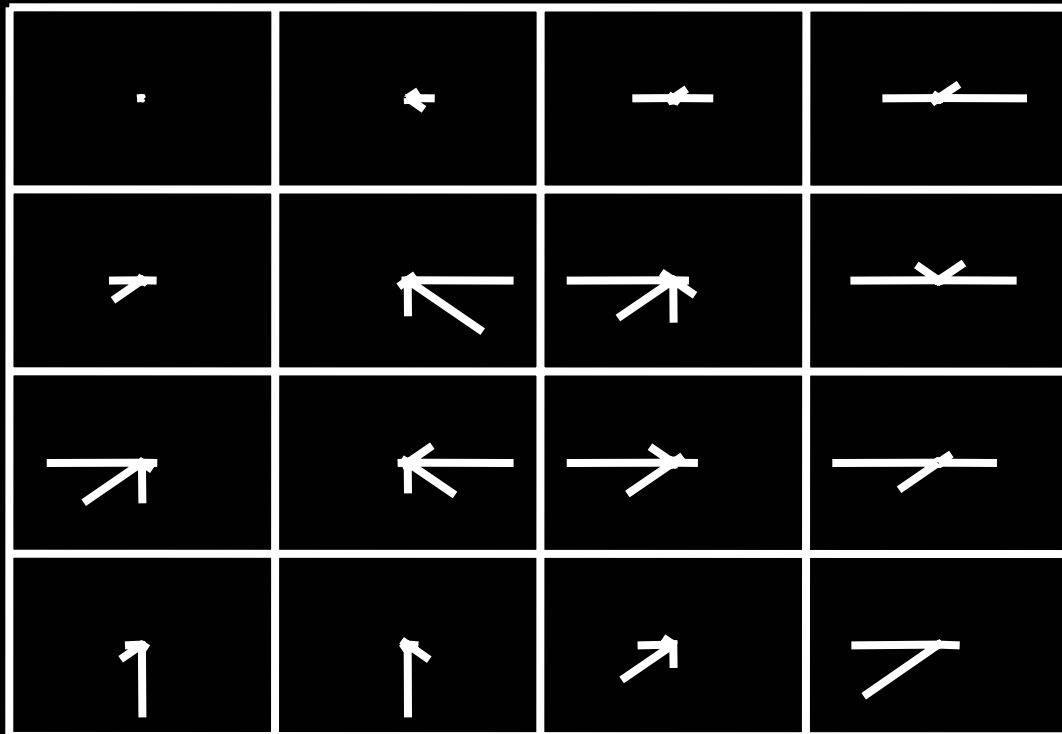
The Earth Mover's Distance Complexity Zoo

- L_1 Manhattan grids $O(N^2 \log N (D + \log N))$
 - Ling and Okada 07



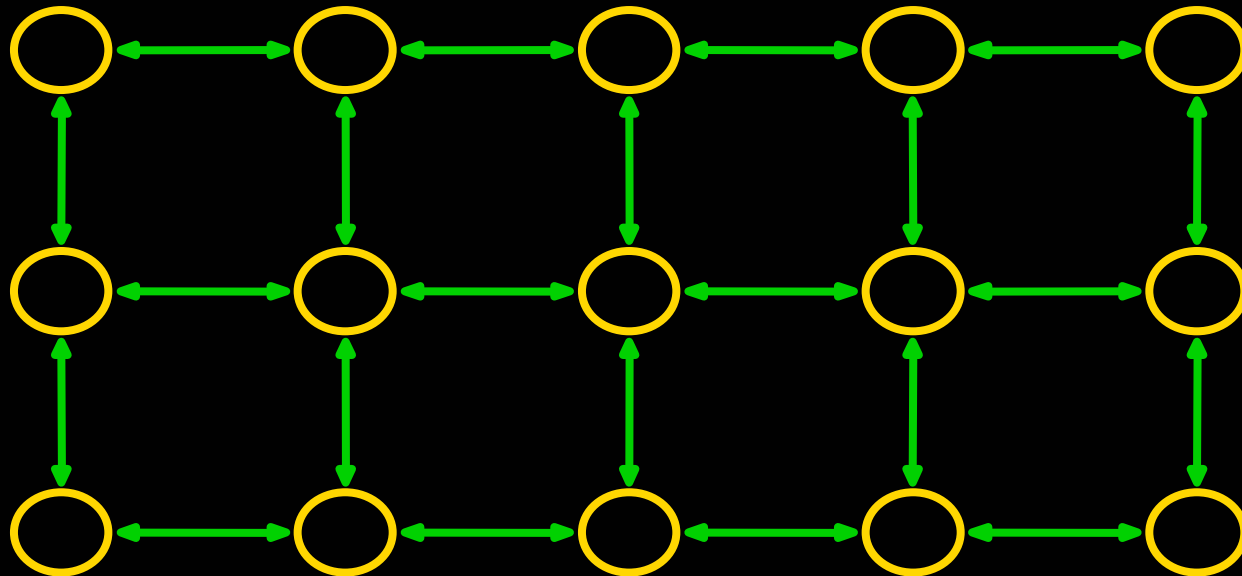
The Earth Mover's Distance Complexity Zoo

- What about N-dimensional histograms with a cyclic dimensions?



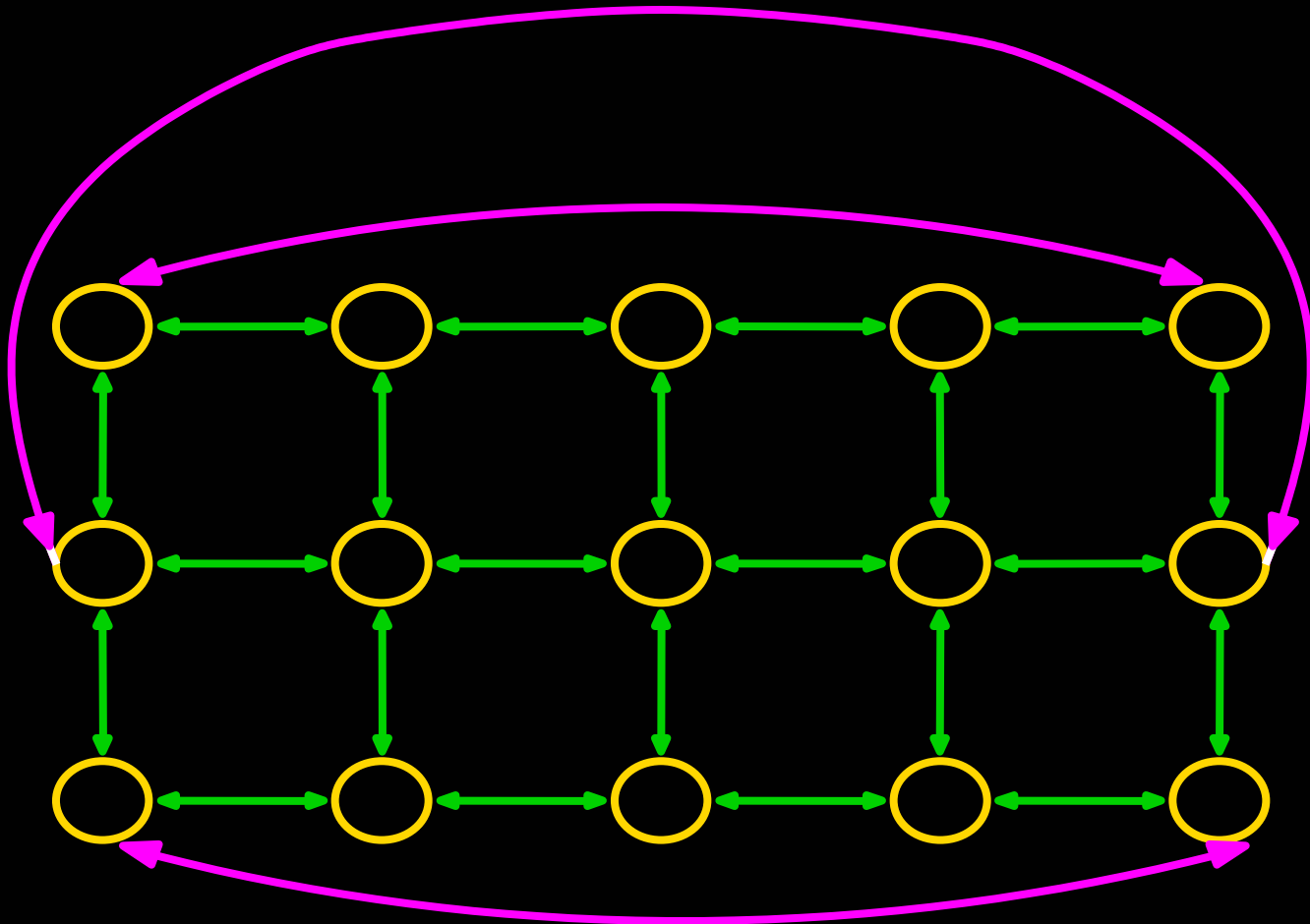
The Earth Mover's Distance Complexity Zoo

- What about N-dimensional histograms with a cyclic dimensions?



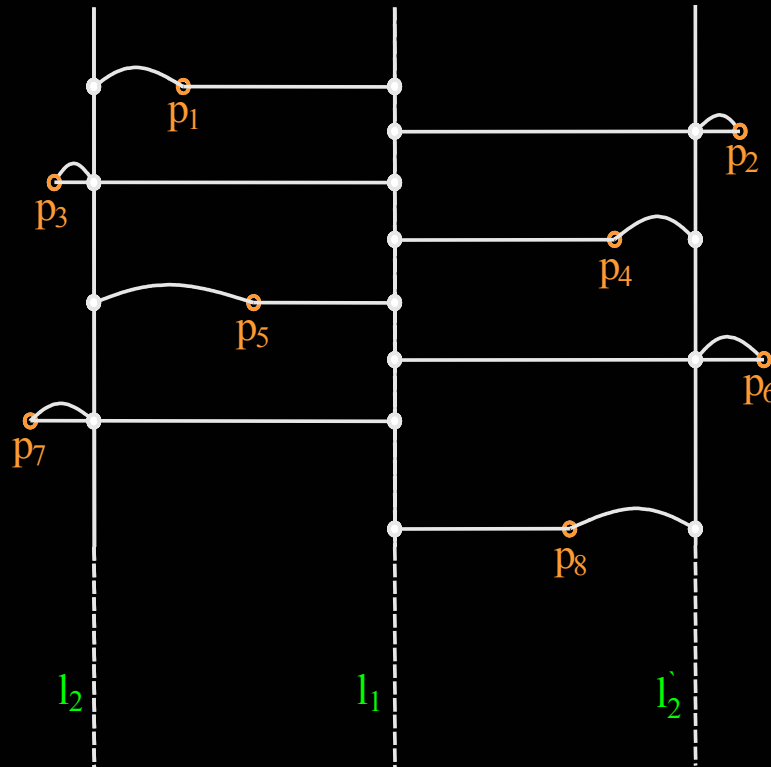
The Earth Mover's Distance Complexity Zoo

- What about N-dimensional histograms with a cyclic dimensions?



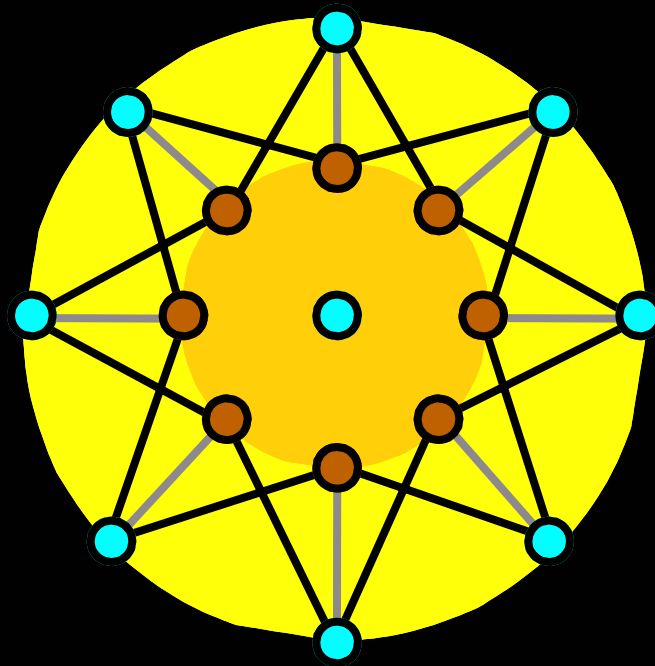
The Earth Mover's Distance Complexity Zoo

- L_1 general histograms $O(N^2 \log^{2D-1} N)$
 - Gudmundsson, Klein, Knauer and Smid 07



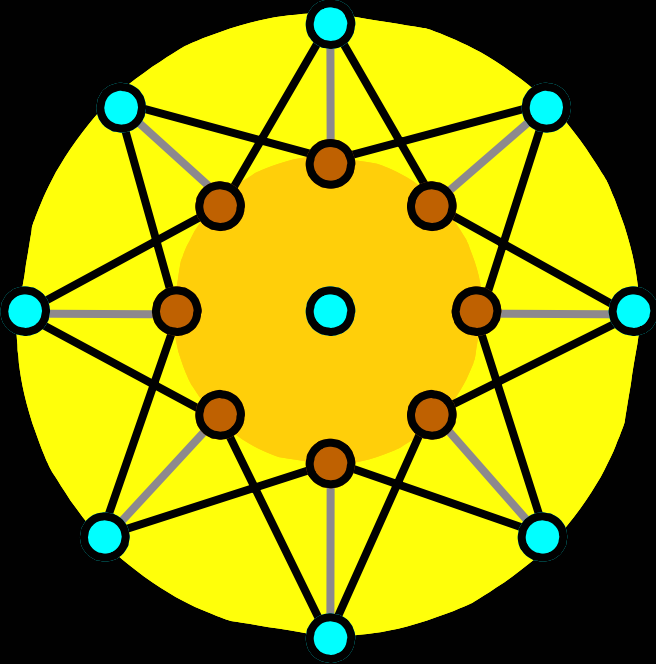
The Earth Mover's Distance Complexity Zoo

- $\min(L_1, 2)$ 1D linear/cyclic $\{1, 2, \dots, \Delta\}$ $O(N)$ histograms
 - Pele and Werman 08



The Earth Mover's Distance Complexity Zoo

- $\min(L_1, 2)$ 1D linear/cyclic $\{1, 2, \dots, \Delta\}$ $O(N)$ histograms
 - Pele and Werman 08



.	←	→	↔
↗	↖↗	↖↗	↖↗↗
↗↗	↖	↗	↖↗
↑	↑	↗	↗

The Earth Mover's Distance Complexity Zoo

- $\min(L_1, 2)$ general $\{1, 2, \dots, \Delta\}^D$ histograms

K is the number of edges with cost 1 $O(N^2 K \log(\frac{N}{K}))$

— Pele and Werman 08

The Earth Mover's Distance Complexity Zoo

- Any thresholded distance $O(N^2 \log N (K + \log N))$
 - Pele and Werman 09

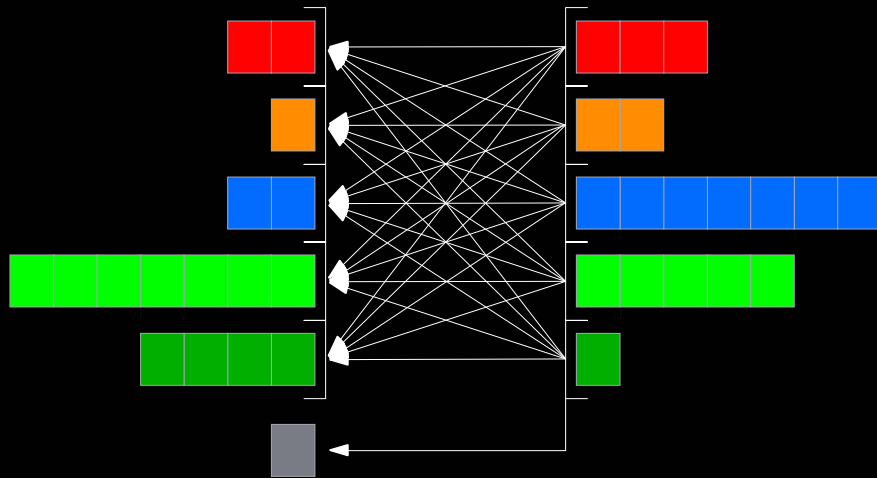
number of edges with cost
different from the threshold

$$O(N^2 \log^2 N) \longleftarrow K = O(\log N)$$

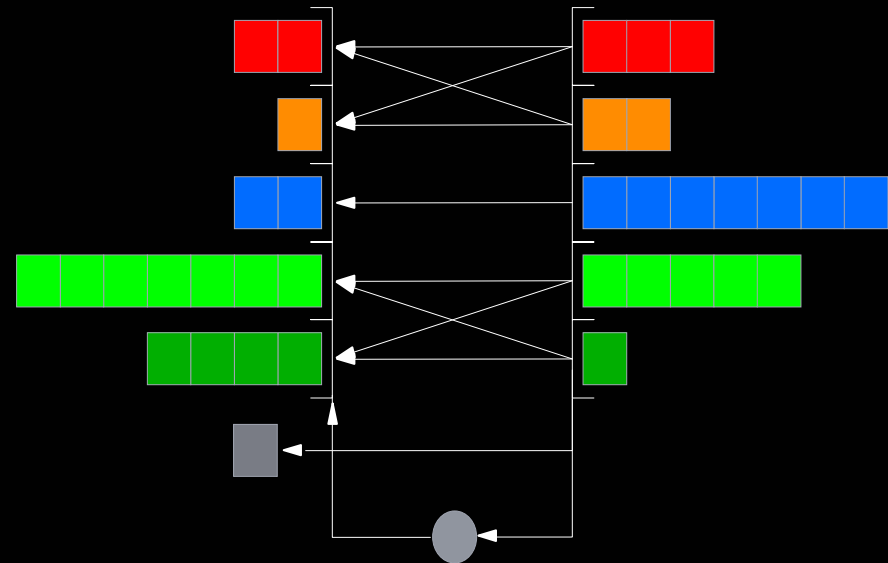
Thresholded Distances

- EMD with a thresholded ground distance is **not** an approximation of EMD.
- It has better performance.

The Flow Network Transformation

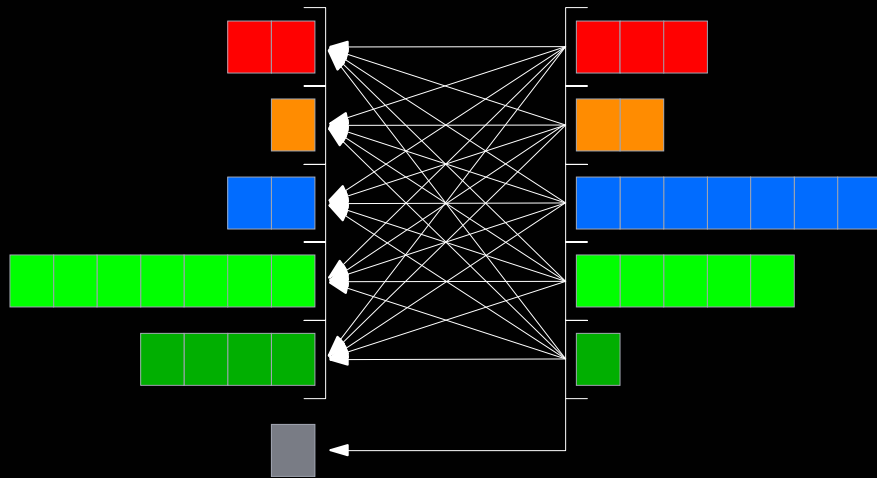


Original
Network

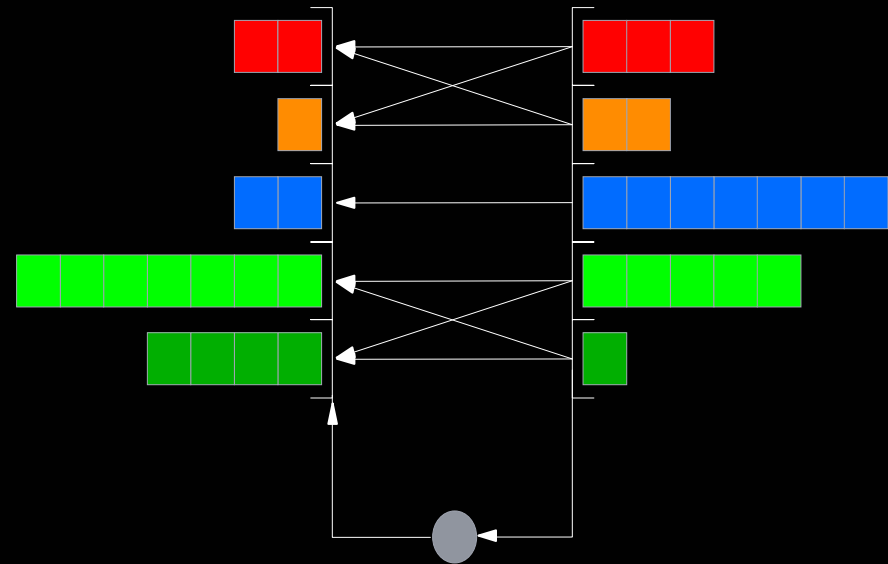


Simplified
Network

The Flow Network Transformation

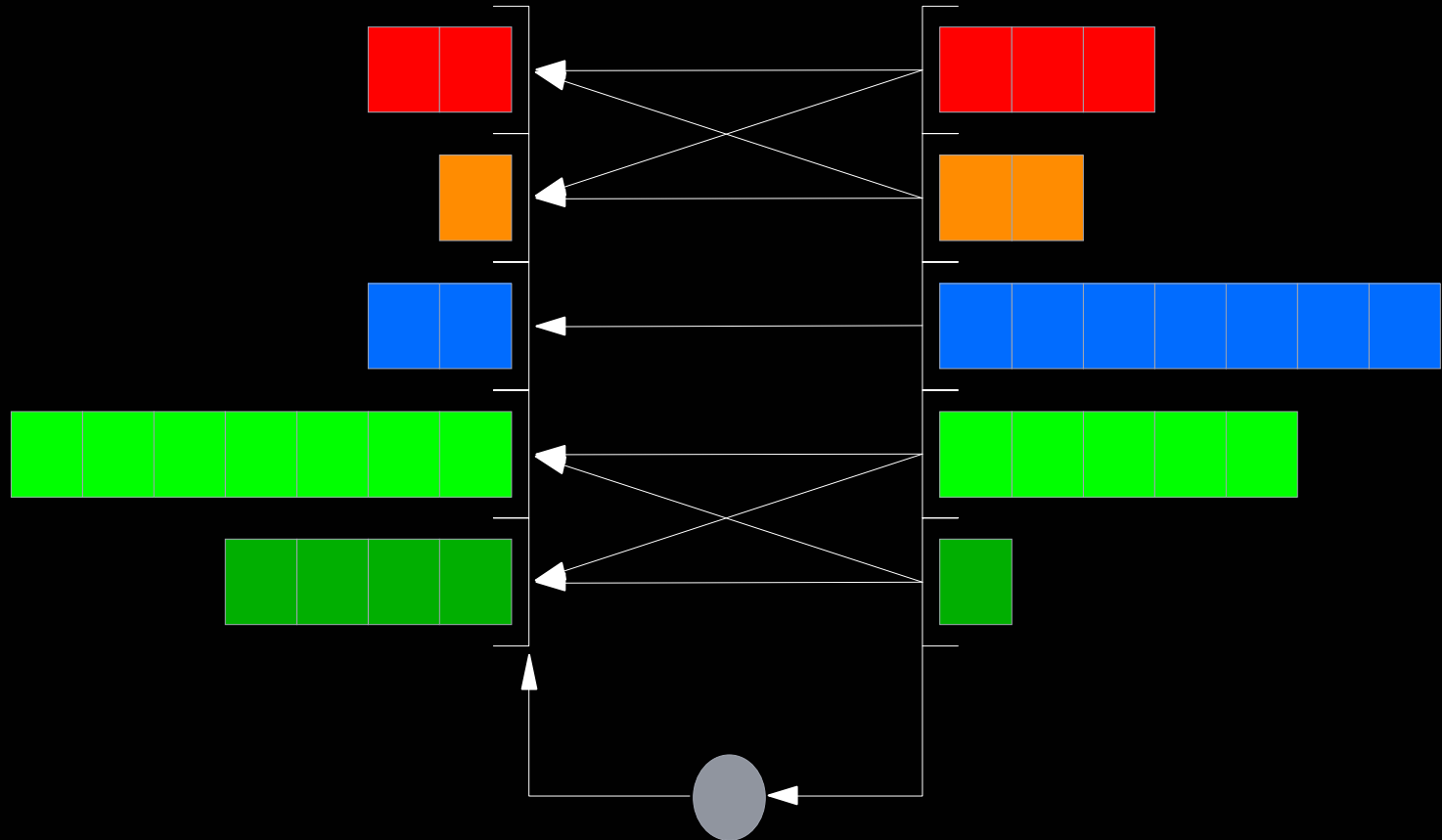


Original
Network



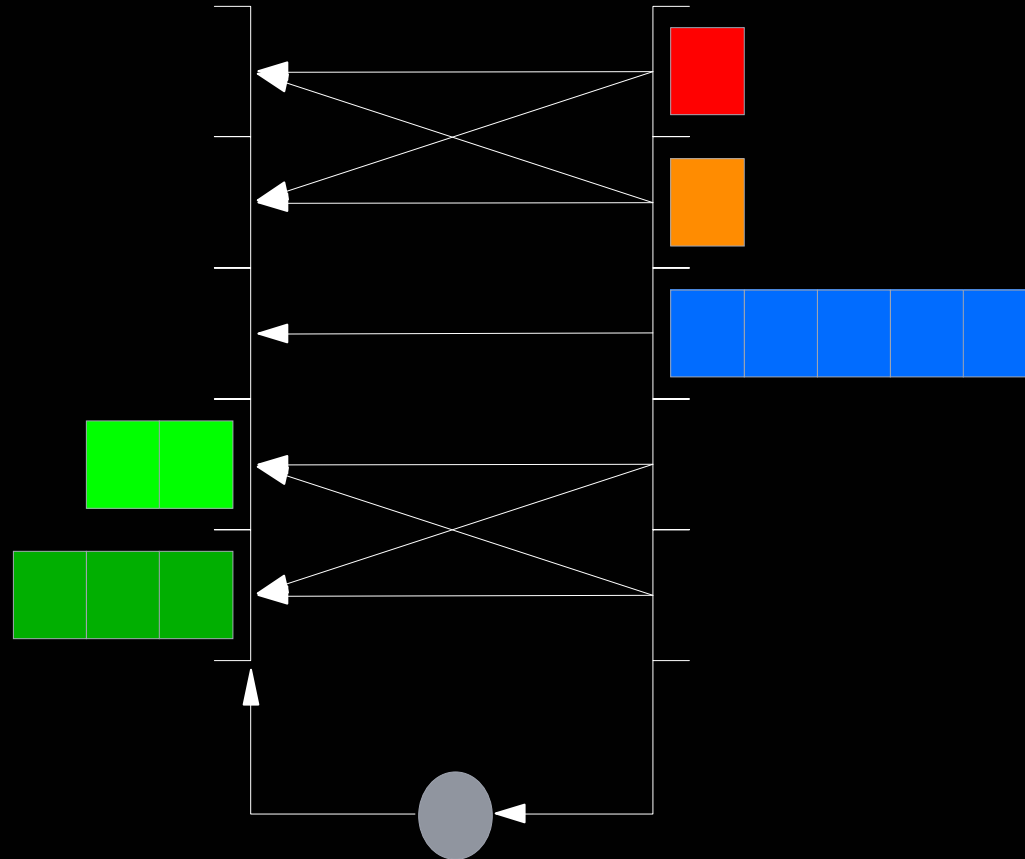
Simplified
Network

The Flow Network Transformation



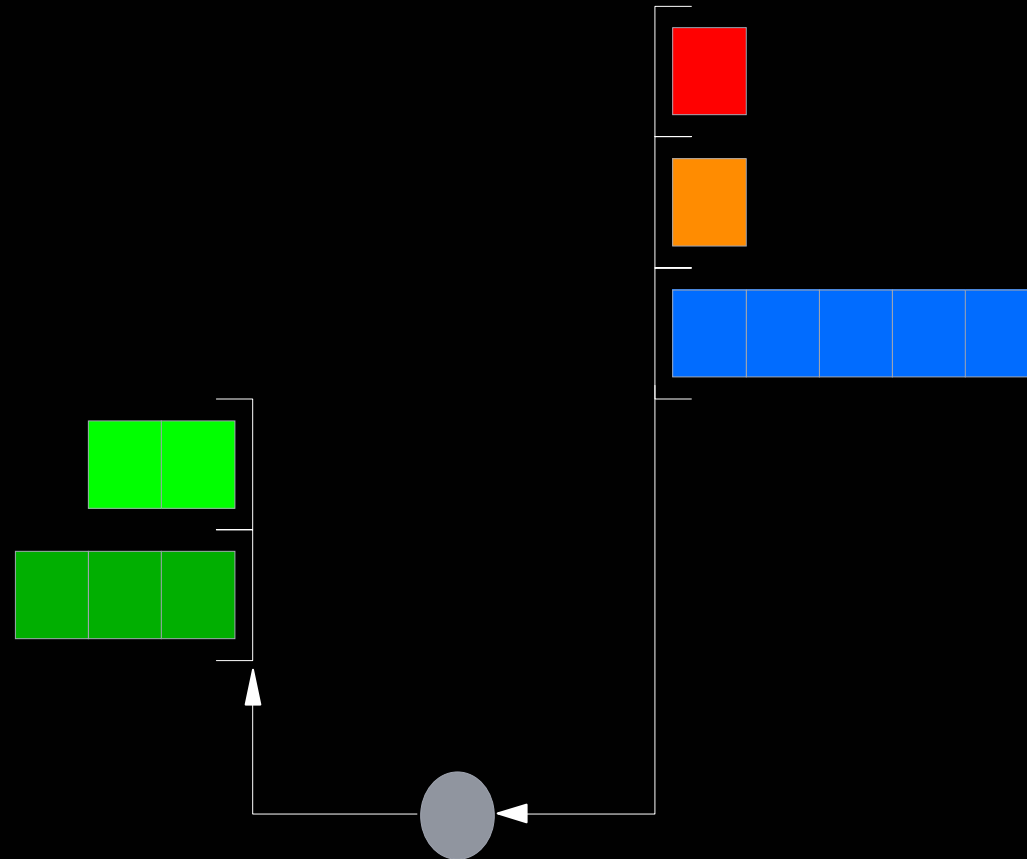
**Flowing the Monge sequence
(if ground distance is a metric,
zero-cost edges are a Monge sequence)**

The Flow Network Transformation



**Removing Empty Bins
and their edges**

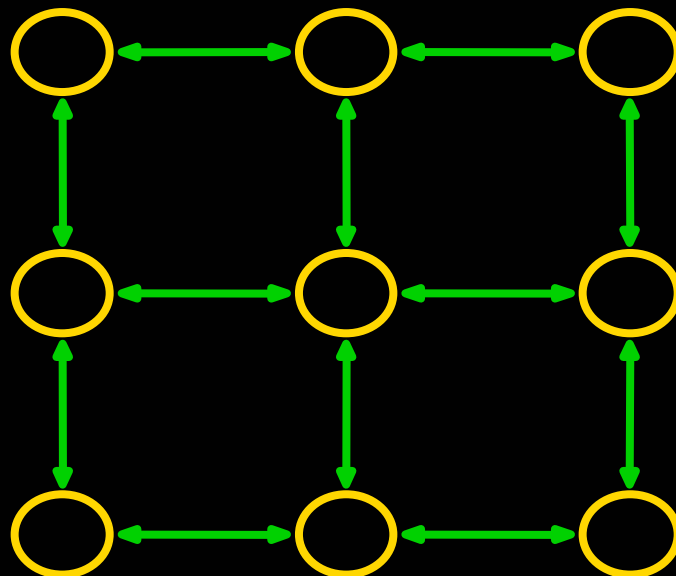
The Flow Network Transformation



We actually finished here....

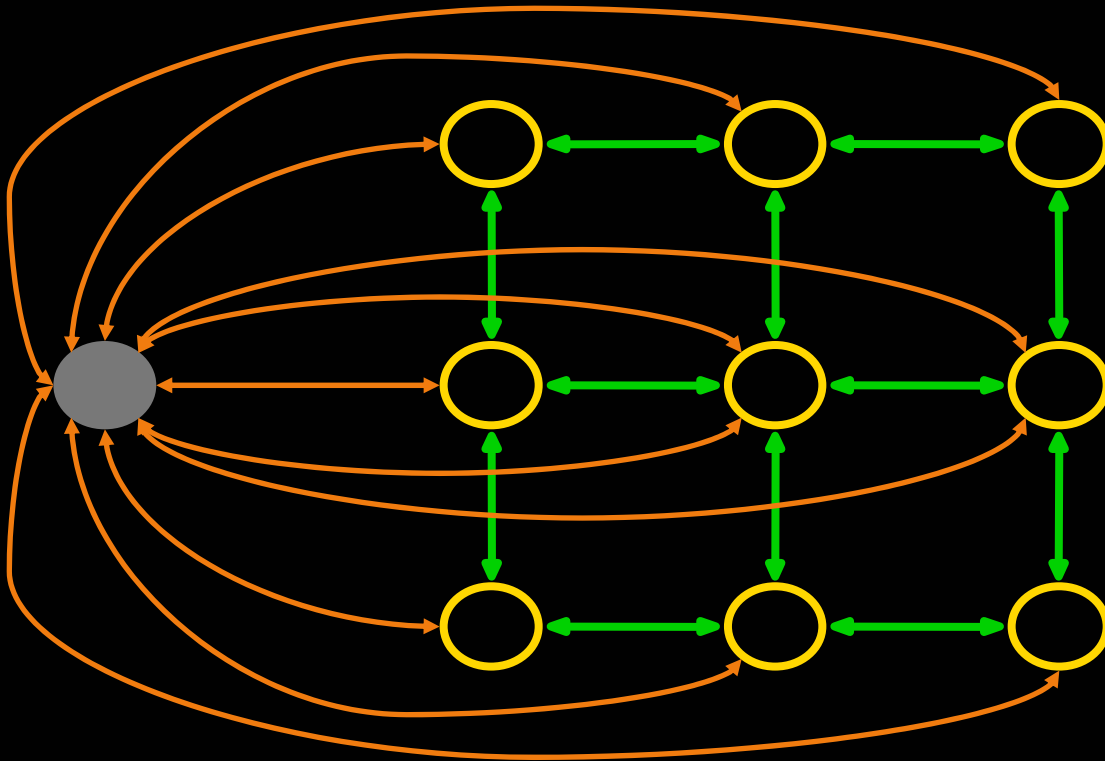
Combining Algorithms

- EMD algorithms can be combined.
- For example L_1 :



Combining Algorithms

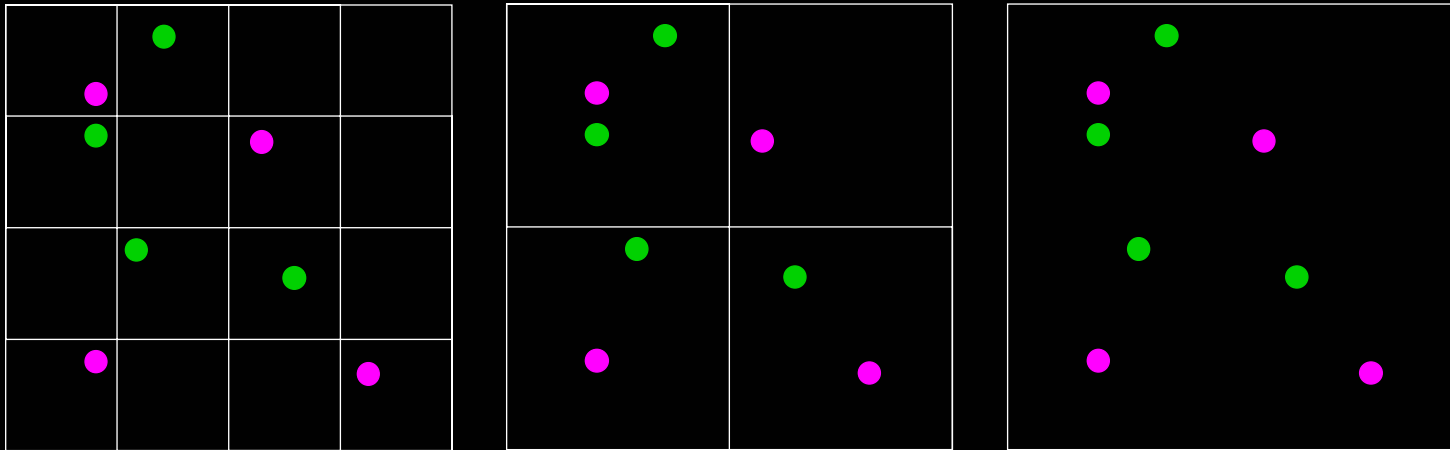
- EMD algorithms can be combined.
- For example, **thresholded** L_1 :



The Earth Mover's Distance Approximations

The Earth Mover's Distance Approximations

- Charikar 02, Indyk and Thaper 03 – approximated EMD on $\{1, \dots, \Delta\}^d$ by embedding it into the L_1 norm.



Time complexity: $O(TNd \log \Delta)$

Distortion (in expectation): $O(d \log \Delta)$

The Earth Mover's Distance Approximations

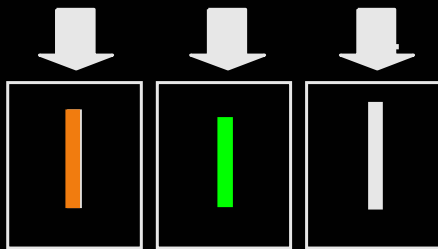
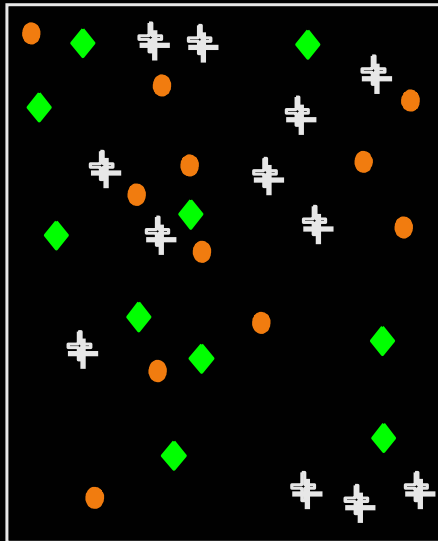
- Grauman and Darrell 05 – Pyramid Match Kernel (PMK) same as Indyk and Thaper, replacing L_1 with **histogram intersection**.
- PMK approximates EMD with partial matching.
- PMK is a mercer kernel.
- Time complexity & distortion – same as Indyk and Thaper (proved in Grauman and Darrell 07).

The Earth Mover's Distance Approximations

- Lazebnik, Schmid and Ponce 06 – used PMK in the spatial domain (SPM).

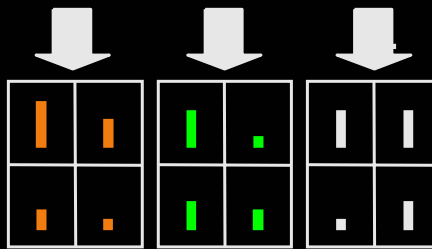
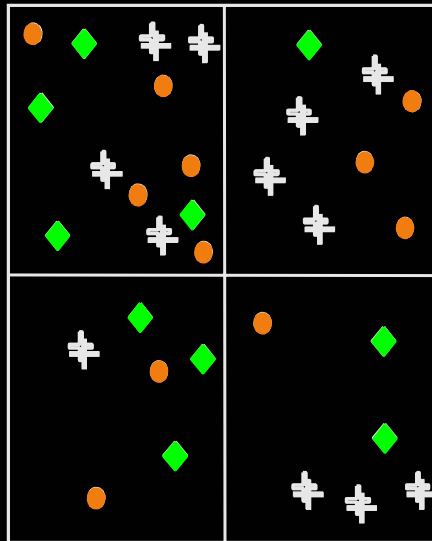
The Earth Mover's Distance Approximations

level 0



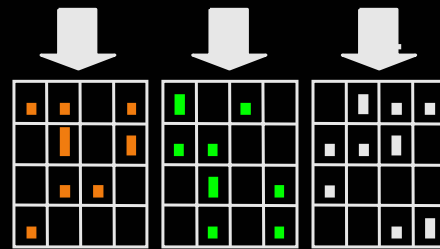
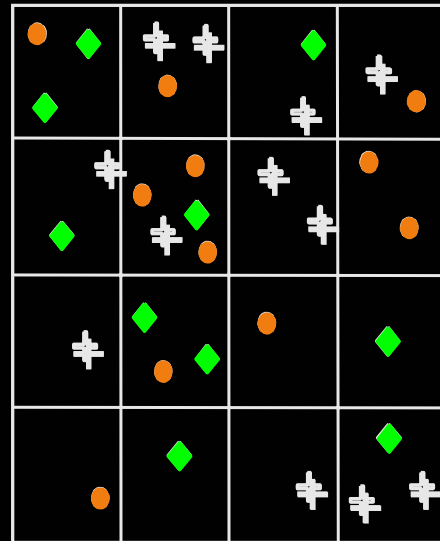
$\times 1/8$

level 1



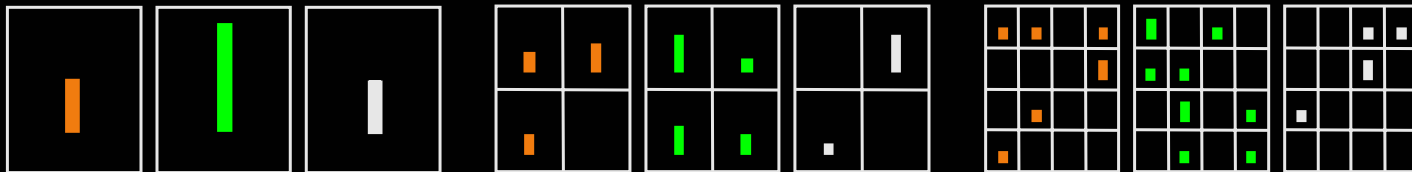
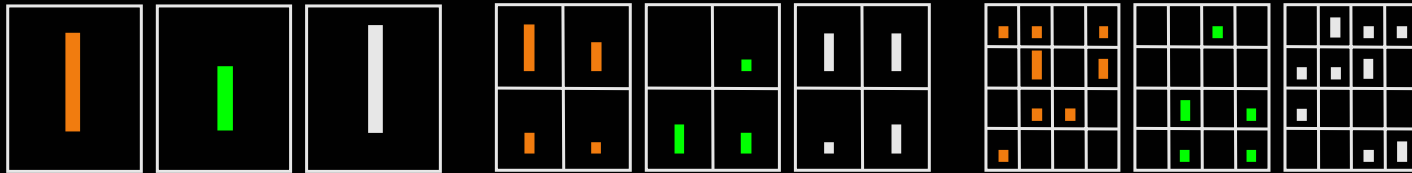
$\times 1/4$

level 2



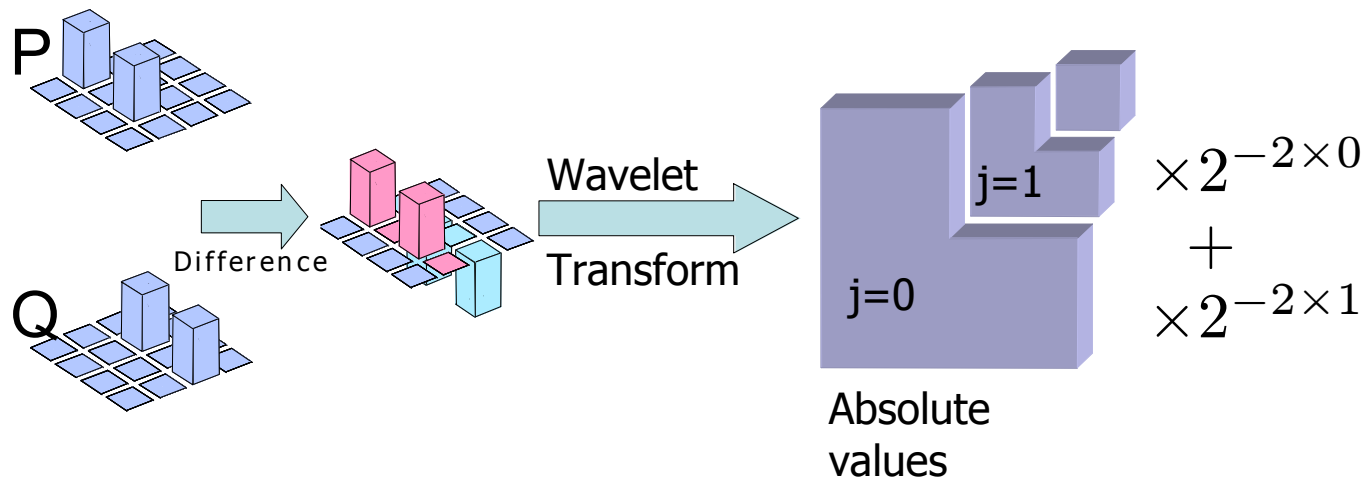
$\times 1/2$

The Earth Mover's Distance Approximations



The Earth Mover's Distance Approximations

- Shirdhonkar and Jacobs 08 - approximated EMD using the sum of absolute values of the weighted wavelet coefficients of the difference histogram.

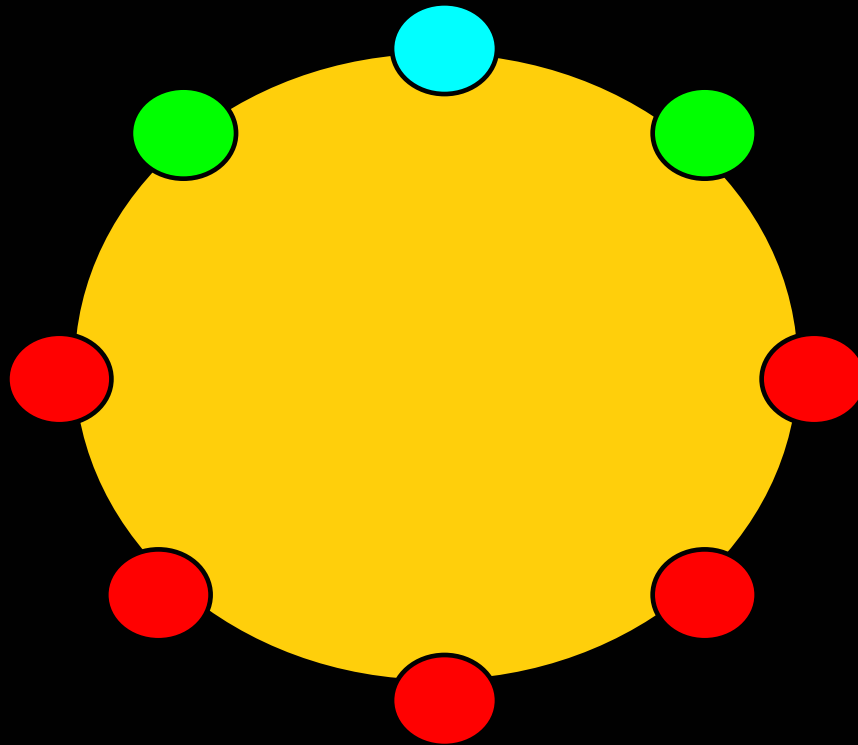


The Earth Mover's Distance Approximations

- Khot and Naor 06 – any embedding of the EMD over the d -dimensional Hamming cube into L_1 must incur a distortion of $\Omega(d)$.
- Andoni, Indyk and Krauthgamer 08 - for sets with cardinalities upper bounded by a parameter s the distortion reduces to $O(\log s \log d)$.
- Naor and Schechtman 07 - any embedding of the EMD over $\{0, 1, \dots, \Delta\}^2$ must incur a distortion of $\Omega(\sqrt{\log \Delta})$.

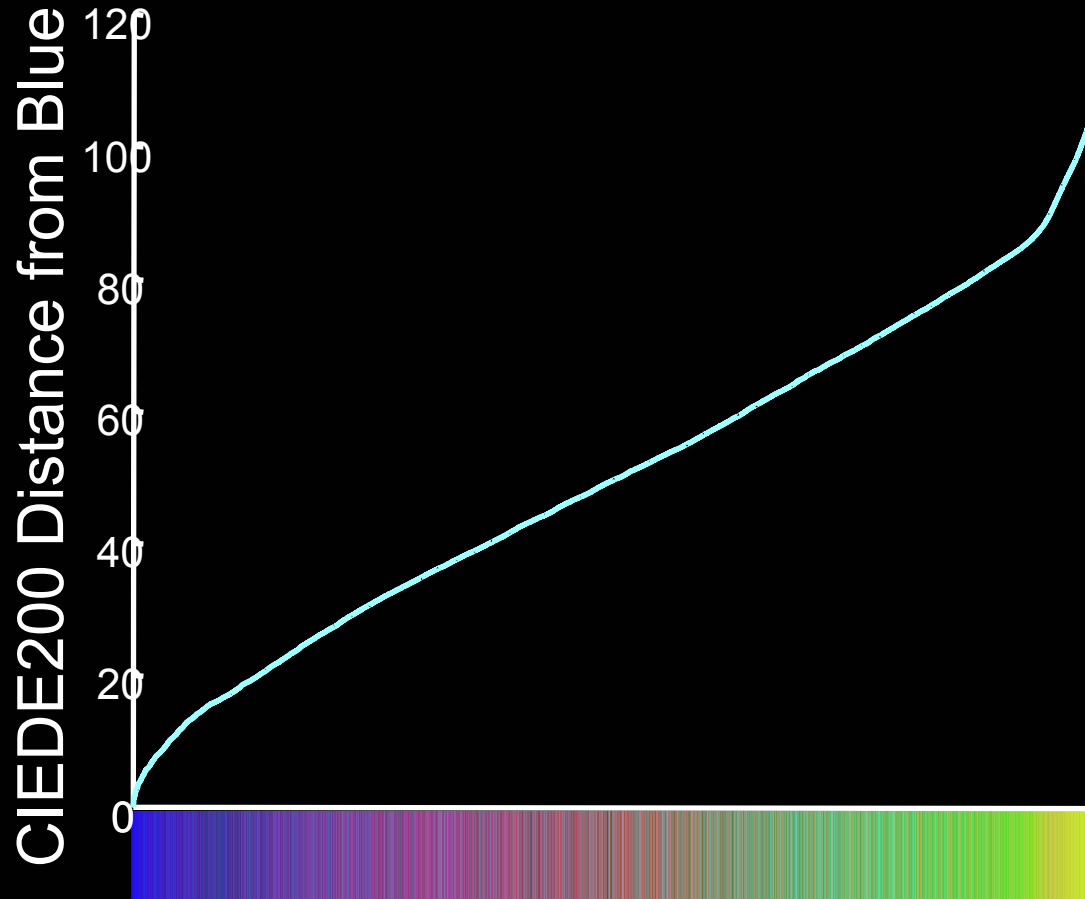
Robust Distances

- Very high distances $\Rightarrow$ outliers $\Rightarrow$ same difference.



Robust Distances

- With colors, the natural choice.



Robust Distances

$$\Delta E_{00}(\text{blue}, \text{red}) = 56$$

$$\Delta E_{00}(\text{blue}, \text{yellow}) = 102$$

Robust Distances - Exponent

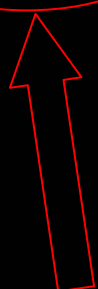
- Usually a negative exponent is used:
 - Let $d(a, b)$ be a distance measure between two features - a, b .
 - The negative exponent distance is:

$$d_e(a, b) = 1 - e^{\frac{-d(a, b)}{\sigma}}$$

Robust Distances - Exponent

- Exponent is used because (Ruzon and Tomasi 01):
robust, smooth, monotonic, and a metric

Input is always discrete anyway ...



Robust Distances - Thresholded

- Let $d(a, b)$ be a distance measure between two features - a, b .
- The thresholded distance with a threshold of $t > 0$ is:

$$d_t(a, b) = \min(d(a, b), t).$$

Thresholded Distances

- Thresholded metrics are also metrics (Pele and Werman ICCV 2009).
- Better results.
- Pele and Werman ICCV 2009 algorithm computes EMD with thresholded ground distances much faster.
- Thresholded distance corresponds to **sparse similarities matrix** -> faster QC / QF computation.

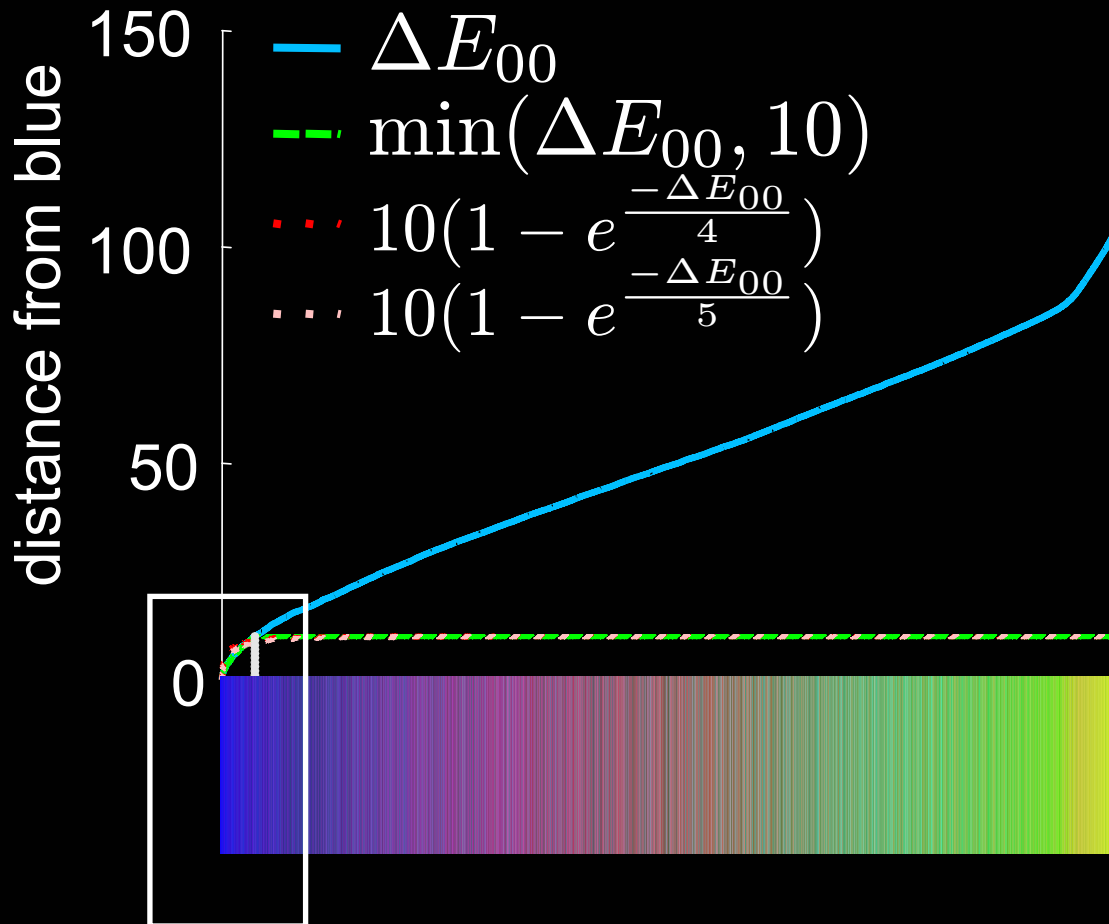
$$A_{ij} = 1 - \frac{D_{ij}}{\max_{ij}(D_{ij})}$$

Thresholded Distances

- Thresholded vs. exponent:
 - Fast computation of cross-bin distances with a thresholded ground distance.
 - Exponent changes small distances – can be a problem (e.g. color differences).

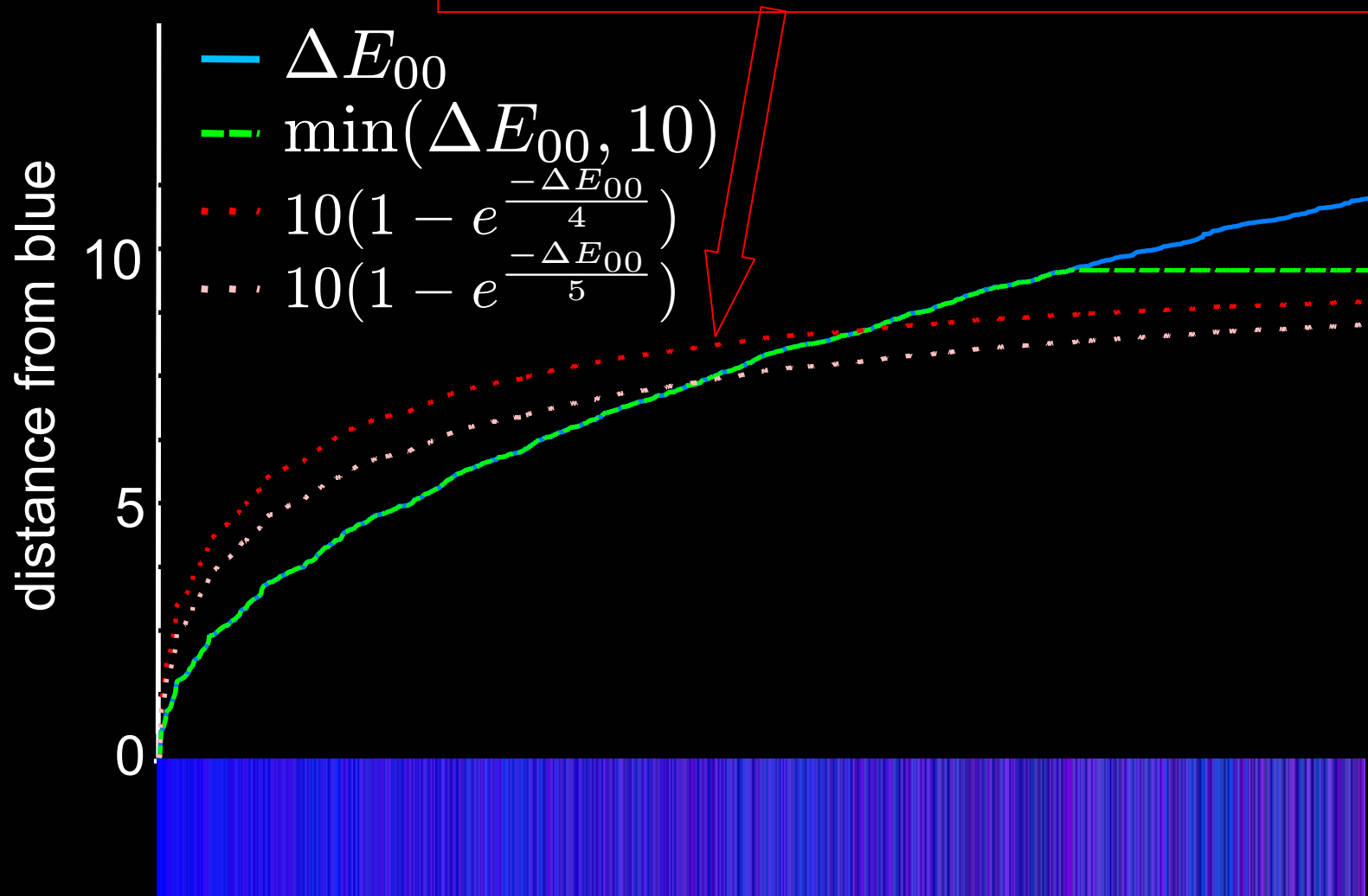
Thresholded Distances

- Color distance should be thresholded (robust).



Thresholded Distances

Exponent changes small distances



A Ground Distance for SIFT

The ground distance between two SIFT bins (x_i, y_i, o_i) and (x_j, y_j, o_j) :

$$d_R = ||(x_i, y_i) - (x_j, y_j)||_2 + \min(|o_i - o_j|, M - |o_i - o_j|)$$

$$d_T = \min(d_R, T)$$

A Ground Distance for Color Image

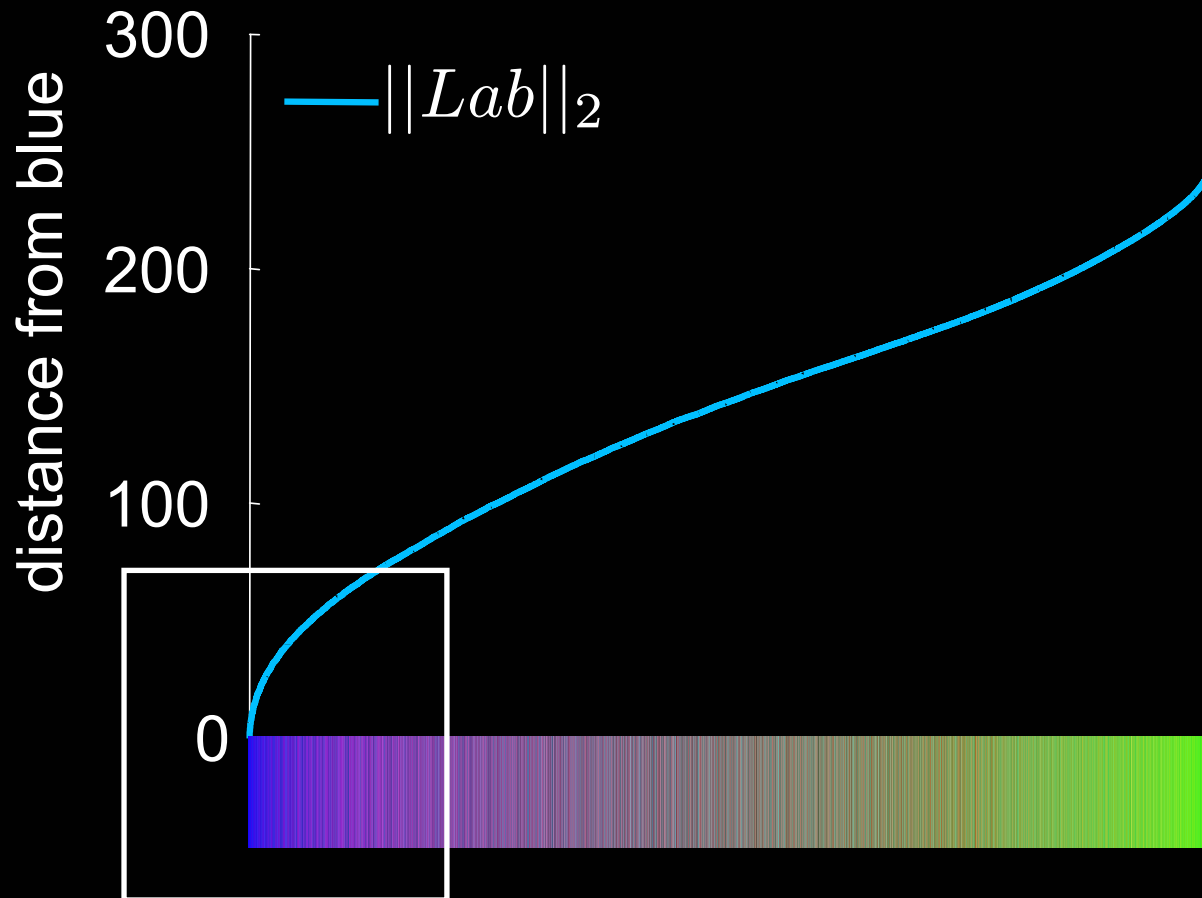
The ground distances between two LAB image bins $(x_i, y_i, L_i, a_i, b_i)$ and $(x_j, y_j, L_j, a_j, b_j)$ we use are:

$$dc_T = \min(||(x_i, y_i) - (x_j, y_j)||_2) + \Delta_{00}((L_i, a_i, b_i), (L_j, a_j, b_j), T)$$

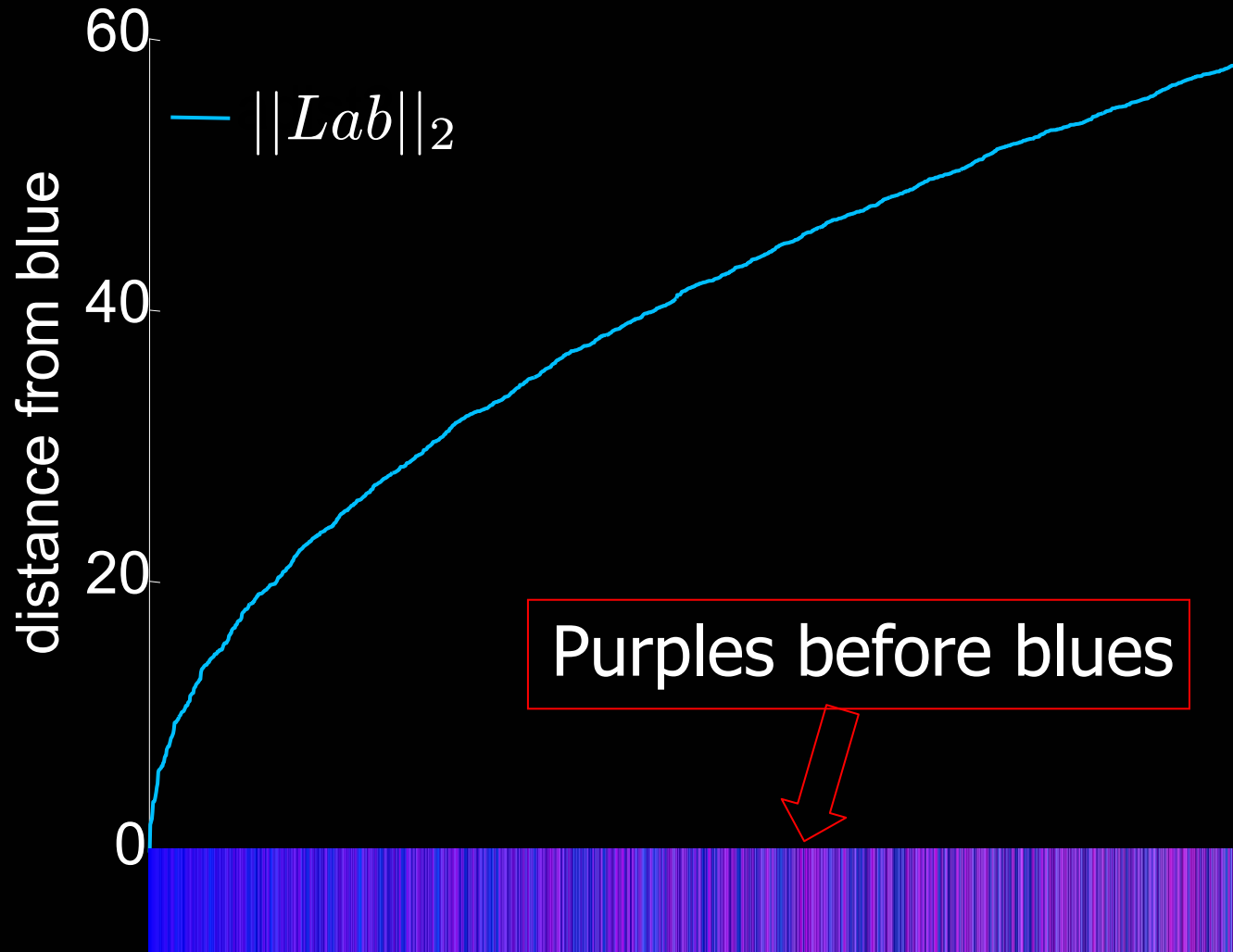
Perceptual Color Differences

Perceptual Color Differences

- Euclidean distance on $L^*a^*b^*$ space is widely considered as perceptual uniform.



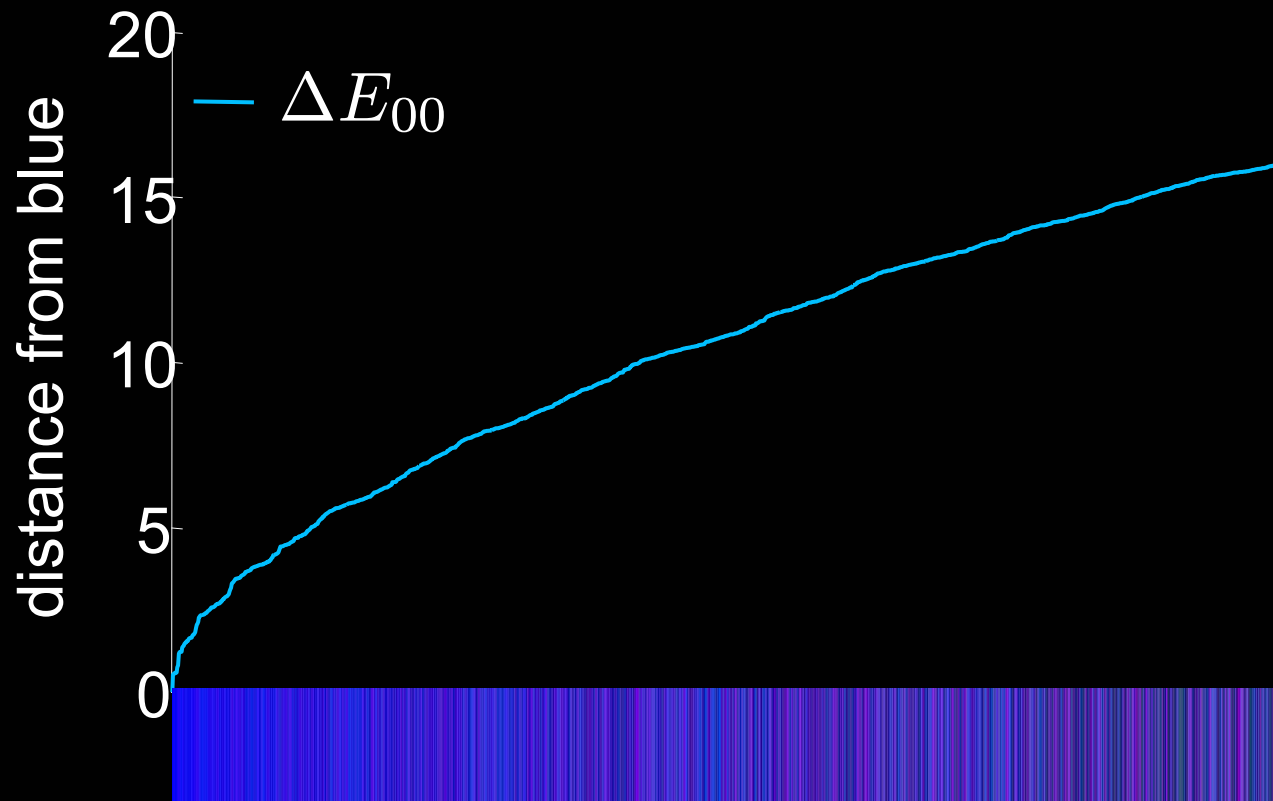
Perceptual Color Differences



Perceptual Color Differences

- ΔE_{00} on L*a*b* space is better.

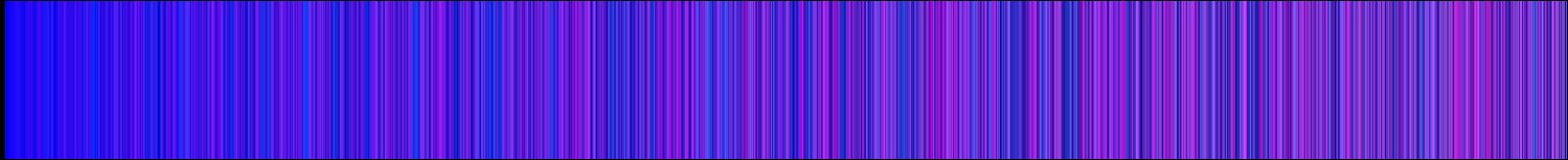
Luo, Cui and Rigg 01. Sharma, Wu and Dalal 05.



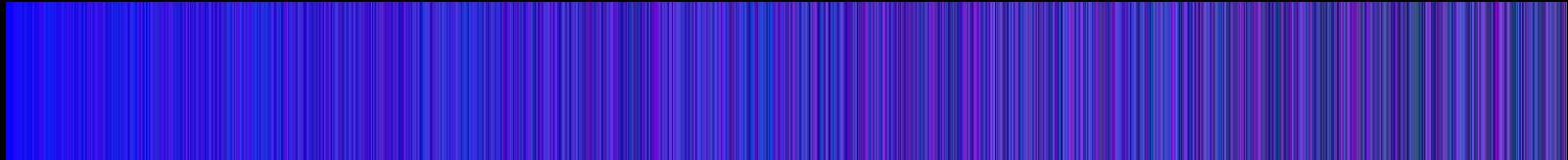
Perceptual Color Differences

- ΔE_{00} on $L^*a^*b^*$ space is better.

$||Lab||_2$

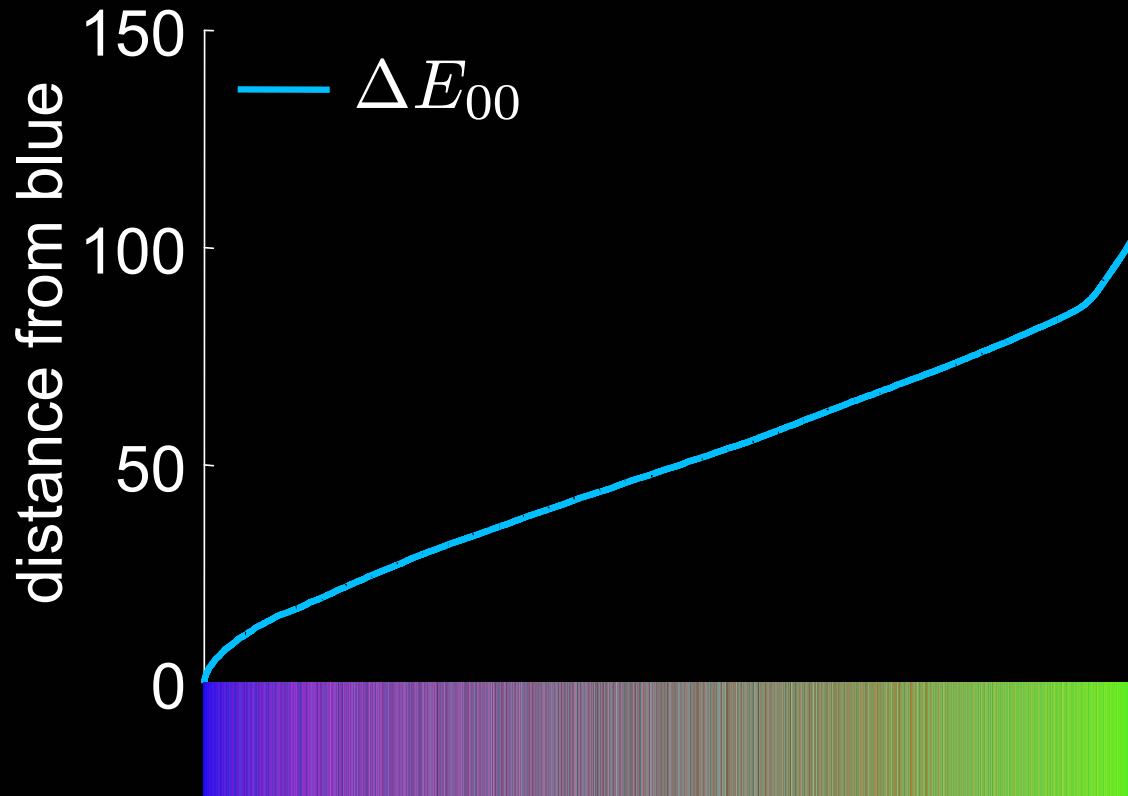


ΔE_{00}



Perceptual Color Differences

- ΔE_{00} on L*a*b* space is better.



Perceptual Color Differences

- ΔE_{00} on L*a*b* space is better.
- But still has major problems.

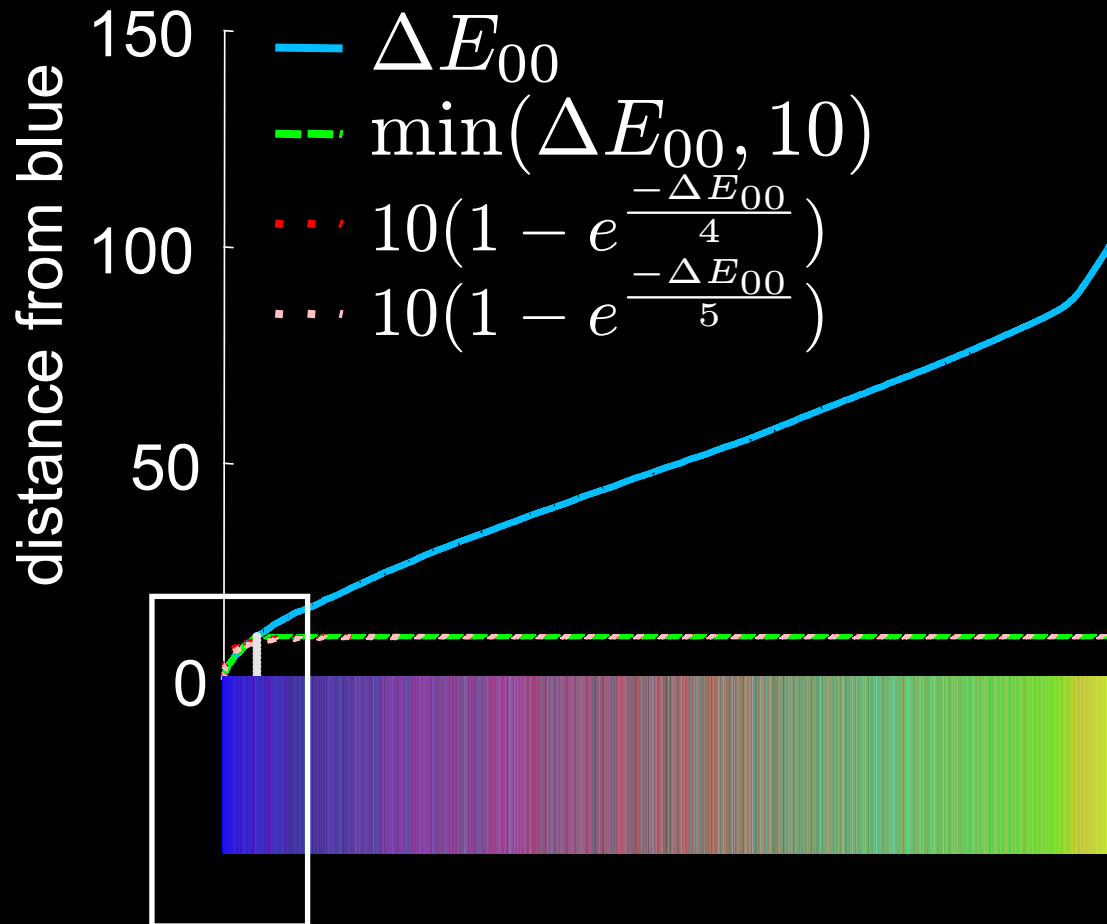
$$\Delta E_{00}(\text{blue}, \text{red}) = 56$$

$$\Delta E_{00}(\text{blue}, \text{yellow}) = 102$$

- Color distance should be thresholded (robust).

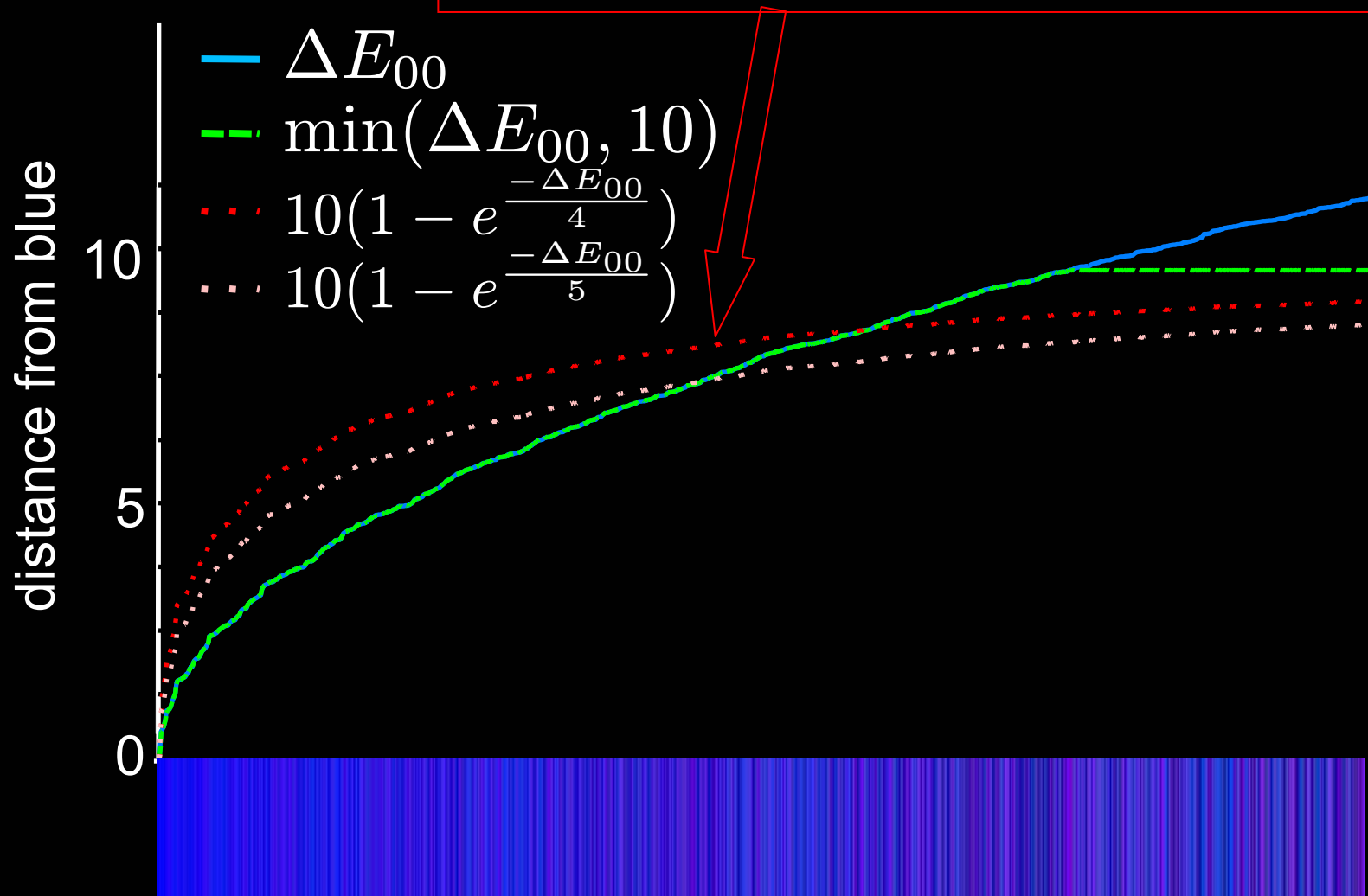
Perceptual Color Differences

- Color distance should be saturated (robust).



Thresholded Distances

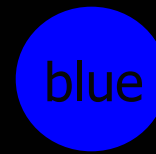
Exponent changes small distances



Perceptual Color Descriptors

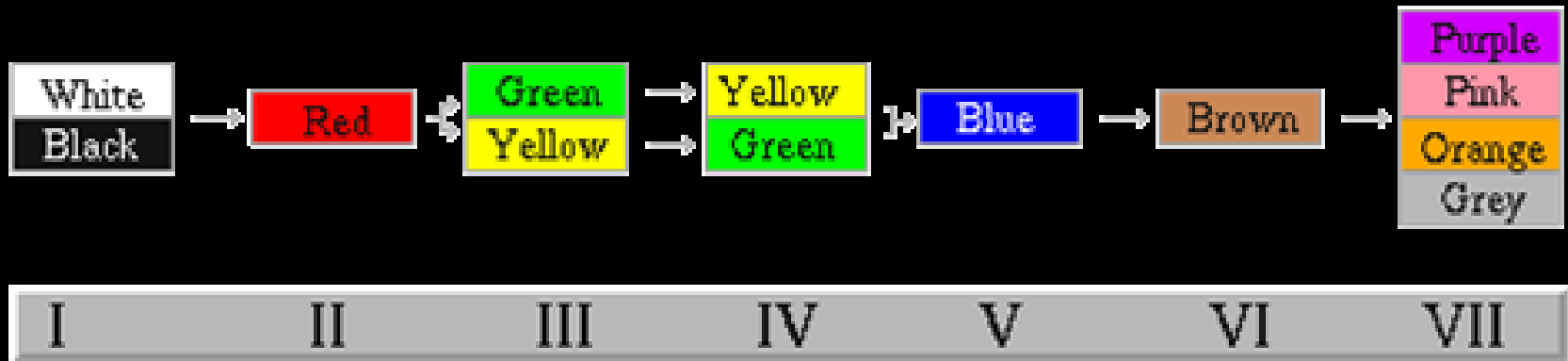
Perceptual Color Descriptors

- 11 basic color terms. Berlin and Kay 69.



Perceptual Color Descriptors

- 11 basic color terms. Berlin and Kay 69.



Perceptual Color Descriptors

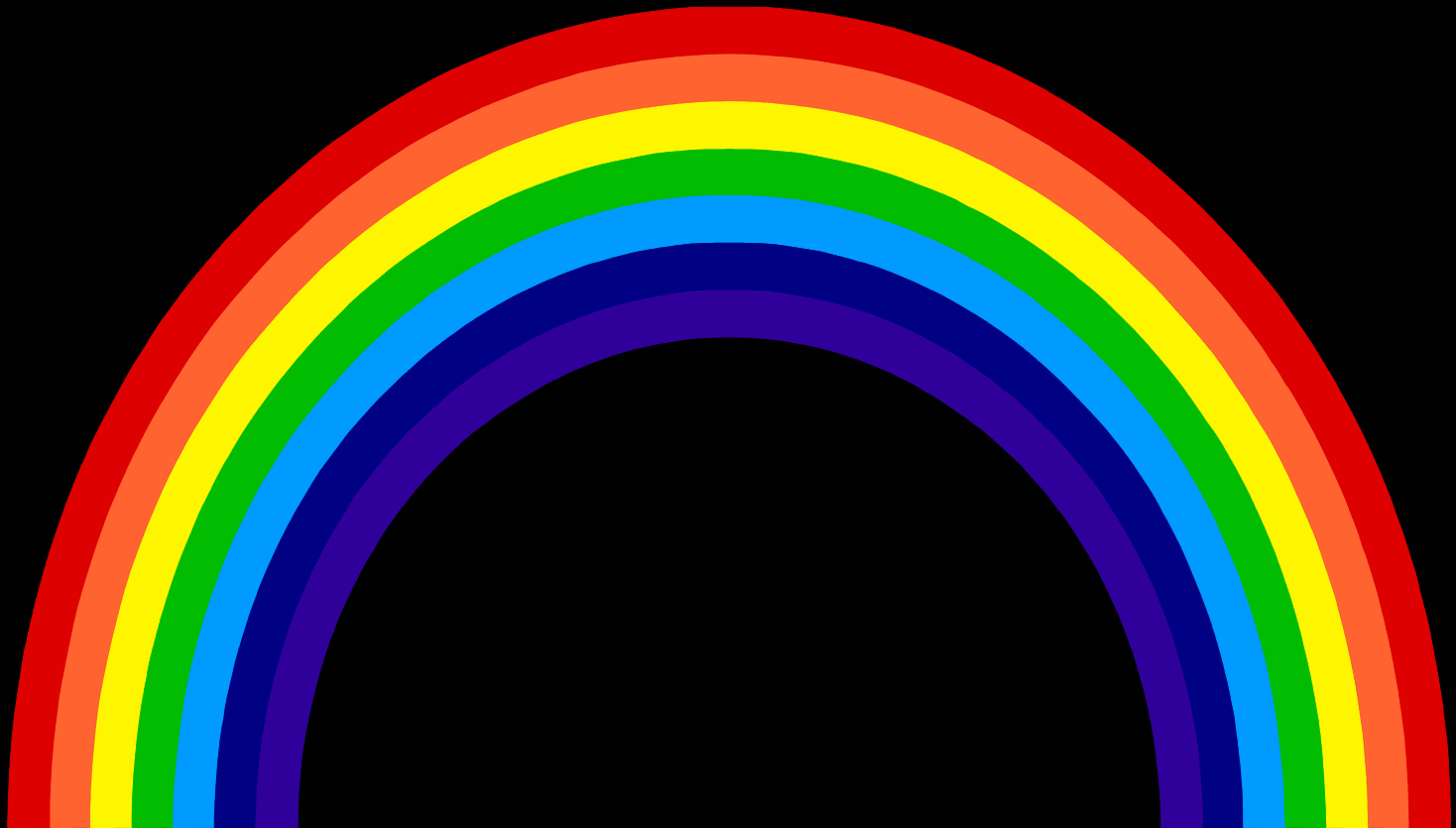
- 11 basic color terms. Berlin and Kay 69.



- Image copyright by Eric Rolph. Taken from:
<http://upload.wikimedia.org/wikipedia/commons/5/5c/Double-alaskan-rainbow.jpg>

Perceptual Color Descriptors

- 11 basic color terms. Berlin and Kay 69.



Perceptual Color Descriptors

- How to give each pixel an “11-colors” description ?

Perceptual Color Descriptors

- Learning Color Names from Real-World Images,
J. van de Weijer, C. Schmid, J. Verbeek CVPR 2007.



Perceptual Color Descriptors

- Learning Color Names from Real-World Images, J. van de Weijer, C. Schmid, J. Verbeek CVPR 2007.

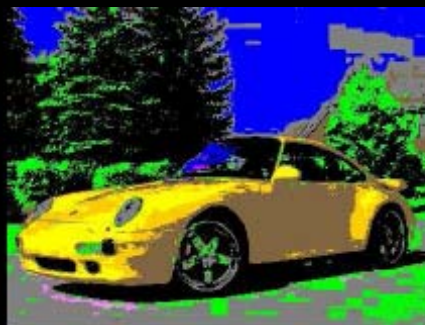


Perceptual Color Descriptors

- Learning Color Names from Real-World Images, J. van de Weijer, C. Schmid, J. Verbeek CVPR 2007.

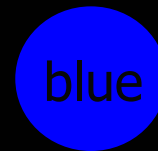
chip-based

real-world



Perceptual Color Descriptors

- Learning Color Names from Real-World Images, J. van de Weijer, C. Schmid, J. Verbeek CVPR 2007.
- For each color returns a probability distribution over the 11 basic colors.



Perceptual Color Descriptors

- Applying Color Names to Image Description, J. van de Weijer, C. Schmid ICIP 2007.
- Outperformed state of the art color descriptors.

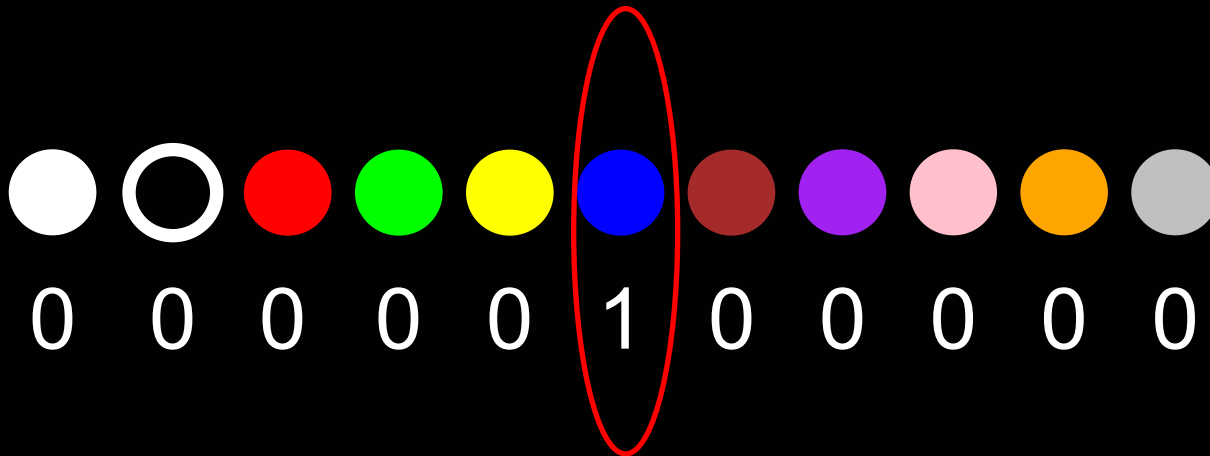


Perceptual Color Descriptors

- Using illumination invariants – black, gray and white are the same.
- “Too much invariance” happens in other cases
(Local features and kernels for classification of texture and object categories:
An in-depth study - Zhang, Marszalek, Lazebnik and Schmid. IJCV 2007,
Learning the discriminative power-invariance trade-off - Varma and Ray. ICCV 2007).
- To conclude: Don't solve imaginary problems.

Perceptual Color Descriptors

- This method is still not perfect.
- 11 color vector for **purple(255,0,255)** is:



- In real world images there are no such over-saturated colors.

Open Questions

- EMD variant that reduces the effect of large bins.
- Learning the ground distance for EMD.
- Learning the similarity matrix and normalization factor for QC.

Hands-On Code Example

<http://www.cs.huji.ac.il/~ofirpele/FastEMD/code/>

<http://www.cs.huji.ac.il/~ofirpele/QC/code/>

Tutorial:

http://www.cs.huji.ac.il/~ofirpele/DFML_ECCV2010_tutorial/

Or  "Ofir Pele"